HEAT TRANSFER COEFFICIENTS AND POWER REQUIREMENTS OF CRYSTALLIZERS WITH EXTENDED SURFACE COOLING ELEMENTS

By E. E. A. ROUILLARD
Sugar Milling Research Institute

Abstract

Heat transfer and power requirements in crystallizers stirred with finned tubes were studied experimentally. Correlations were developed based on dimensionless numbers by least-square regression analysis. The resulting equations have a correlation coefficient of 0,935 and 0,984 respectively for the heat transfer coefficient and power consumption.

Introduction

The theoretical requirements for crystal growth demand that the rate of cooling of the massecuite should be such as to maintain a sufficient supersaturation for sucrose deposition on existing crystals, but should not be so great that new nuclei are formed. As sucrose is deposited on the crystals, supersaturation is maintained in the crystallizer by decreasing the temperature and, at the same time, the rate of crystallization is increased by stirring the massecuite which reduces the thickness of the stagnant film of molasses surrounding the crystal. Stirring also prevents the crystals from gradually settling out of the massecuite.

Although crystallizers cooled and stirred by means of revolving water cooled elements have been used in the sugar industry for the past 70 years, only empirical values for heat transfer and power were available until 1952. In that year the effect of speed of rotation and viscosity on the heat transfer and power consumption was pointed out (Honig[3]). The equations proposed for the massecuite film heat transfer coefficient were as follows:

$$h_m = 0{,}02494(V)^{0{,}43} \qquad (1)$$

and:

$$h_m = 0{,}000581/(\mu)^{0{,}925} \qquad (2)$$

and for the power consumption

$$P = 0{,}05326\ D.A.R.\mu \qquad (3)$$

In 1965 the following equation which included Reynolds and Prandtl numbers and a correction for film viscosity was submitted (Gulyi[2]) for calculating the heat transfer in massecuites:

$$U = 32{,}65(k.\rho^{0{,}27}/De)\ (N_{Re})^{0{,}45}\ (N_{Pr})^{0{,}63}\ (N_{Pr})_s^{-0{,}25} \qquad (4)$$

Three equations were proposed in 1967 for calculating the power requirements of scroll, disc and Fletcher crystallizers respectively (Gromkovskii[1]). These were:

$$P = 4\,735\ (47{,}48 + K) \qquad (5)$$

$$P = 3\,945\ (54{,}82 + K) \qquad (6)$$

$$P = 8\,948\ (42{,}04 + K) \qquad (7)$$

In all these equations massecuites were treated as Newtonian fluids. However, massecuite being a pseudoplastic non-Newtonian fluid does not have a constant viscosity at a given temperature but shows a decrease in viscosity with increasing shear rate, and its viscous properties can be represented by the power law equation (Wilkinson[7]).

$$\tau = K\ (Sr)^n \qquad (8)$$

in which the consistency K is similar to the viscosity and the flow behaviour index n is a measure of the departure from non-Newtonian behaviour.

When dealing with these fluids the Reynolds and Prandtl numbers must be modified so that the viscosity is expressed in terms of consistency and flow behaviour index.

The generalized form of the Reynolds number is

$$N^1_{Re} = \frac{De^n\ V^{2-n}\ \rho}{\dfrac{K}{8}\left[\dfrac{6n+2}{n}\right]^n} \qquad (9)$$

and the generalized form of the Prandtl number is

$$N^1_{Pr} = \frac{1}{8}\left[\frac{CpK}{k}\right]_f \left[\frac{V}{De}\right]^{n-1} \left[\frac{6n+2}{n}\right]^n \qquad (10)$$

As the Nusselt number does not include the viscosity, it is unchanged and is expressed as:

$$N_{Nu} = \frac{hDe}{k} \qquad (11)$$

The purpose of the present investigation was to establish equations for predicting the heat transfer coefficients and power consumption using the dimensionless groups modified for non-Newtonian power law fluids.

Experimental Procedure

The investigation was done in the experimental apparatus shown in Fig. 1 and Fig. 2. It consisted of a revolving circular trough driven by a variable speed hydraulic motor. The trough had the following dimensions: internal diameter 0,53 m, external diameter 1,1 m; depth 0,55 m. In it were immersed four stationary cooling elements one of which was mounted on roller bearings and connected to a spring balance from which measurements could be made. Four different cooling elements with a pitch of 27,4:38,6:52,8 and 69,3 mm were tested in this study. Details of the elements are given in Fig. 3.

All temperatures were measured by mercury thermometers with a probable accuracy of $\pm$ 0,5°C. Flow rates of water were obtained by means of a rotameter.

Both B and C massecuites were used in this investigation.

Their consistency and flow behaviour index were determined from shear-stress versus shear-rate data taken with a Brookfield HBT Synchro-lectric viscometer equipped with cylindrical spindles.

The physical properties of massecuite were found in the literature. The heat capacity was calculated from the brix (Hugot[5]).

$$C_{pm} = (1 - 0{,}007\%\ Bx) \times 4\,187 \qquad (12)$$

The thermal conductivity of the system sucrose-water (Honig[4]) was extrapolated to cover the range from 90 to 100 brix.

In a typical run the massecuite was heated to about 65°C by circulating hot water through the heat exchange elements. Cold water was then circulated in the elements and inlet and outlet temperature and flow rate of the water were observed together with the massecuite temperature, dynamometer scale reading and rotating speed of the apparatus. These observations were repeated at speeds varying between 0,038 and 0,0014 m/s and temperatures between 65 and 30°C.

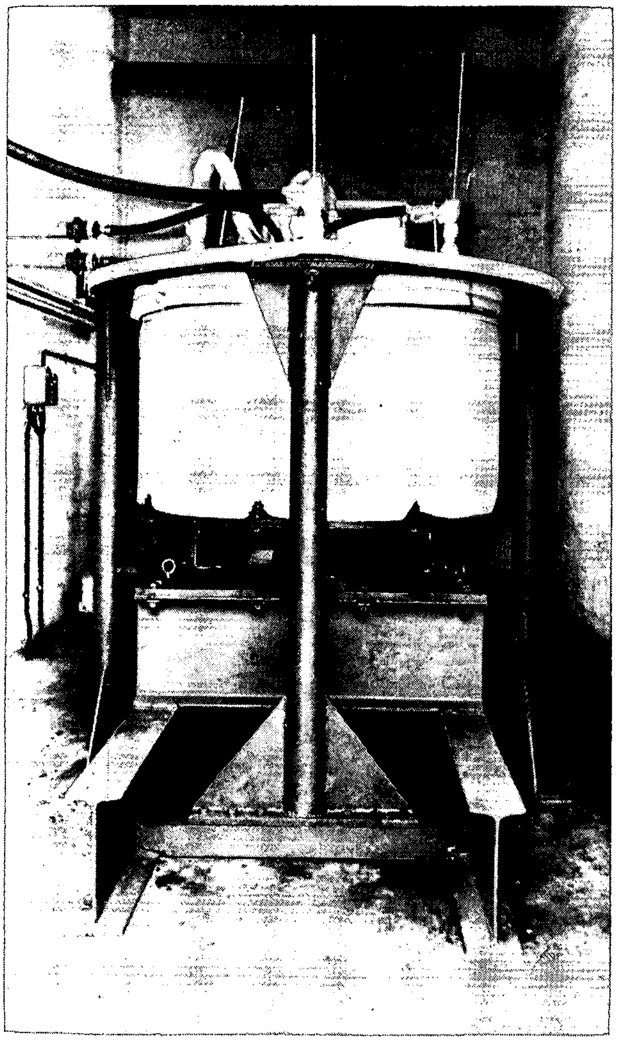


FIGURE 1

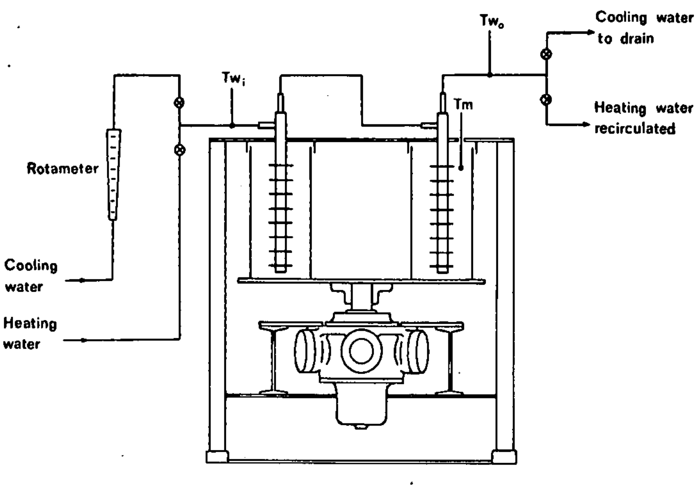


FIGURE 2 Experimental apparatus

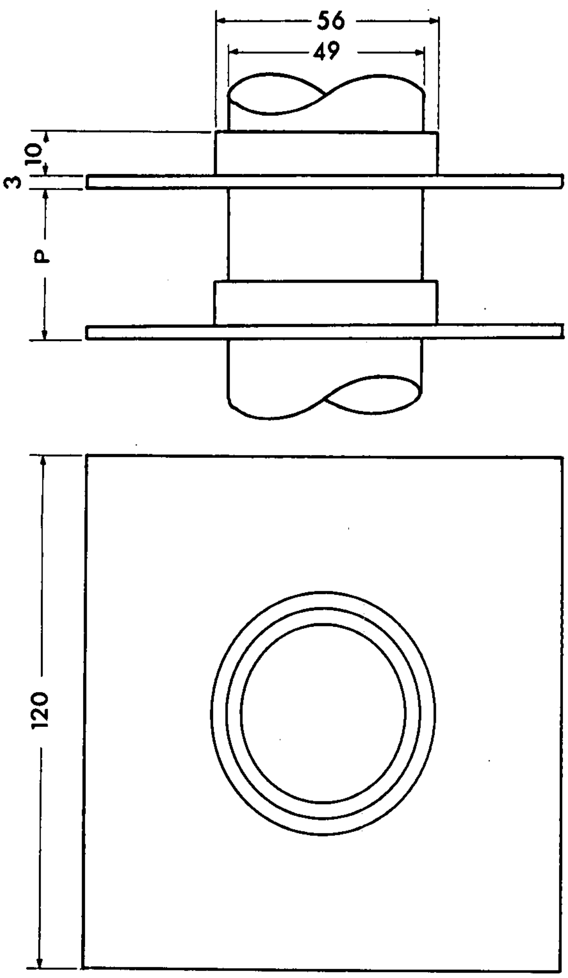


FIGURE 3 Details of finned tubes.

Results

Overall heat transfer coefficient

For the four cooling elements used in this study 88 data points were obtained. The overall heat transfer coefficients were calculated for each point using the equation

$$U = W_w.Cp_w.(t_{wo} - t_{wi}) / A.\Delta t_{lm} \tag{13}$$

The overall resistance to heat flow, 1/u, is equal to the sum of the massecuite film resistance, the scale resistance, the tube wall resistance and the water film resistance; but the resistance of the massecuite film being much larger it was assumed that $U \simeq h_m$.

The mean hydraulic diameter, De, of the cooling elements was obtained from the relationship

$$De = \frac{4 \times \text{Volume of channels between fins}}{\text{wetted surface}} \tag{14}$$

The generalized Reynolds and Prandtl numbers, the Nusselt number, the ratio of the consistency of the bulk of the massecuite to that of the film and the ratio of the mean hydraulic diameter to the width of the fin was calculated for each of the data points. The results for the typical data of Table 1 are listed in Table 2, and all the results are shown graphically in Fig. 4.

Using regression analysis the following equation was obtained for the overall heat transfer coefficient.

$$(N_{Nu})_f = 5{,}12 \times 10^5 (N^1_{Re})^{0{,}0211} (N^1_{Pr})_f^{-0{,}481} (k/k_f)^{0{,}125} (De/F)^{2{,}74} (n)^{6{,}6} \tag{15}$$

The dimensionless numbers used in the regression were the Reynolds and Prandtl numbers, the consistency ratio, the

TABLE 1

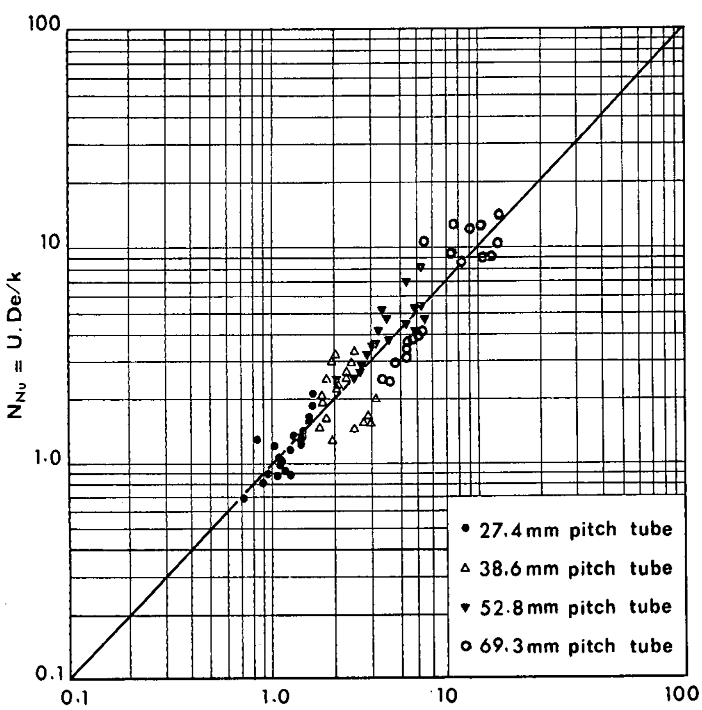
Data for determination of heat transfer coefficient

Velocity ($m\ s^{-1}$)	Hydraulic Diameter (m)	Heat transfer area (m^2)	Flow rate water ($kg\ s^{-1}$)	Temperature (°C) Water inlet	Water outlet	Massecuite
27,4 mm pitch tube						
$K = 1{,}382 \times 10^{-9} e^{4016/Tm}$						
0,03179	0,03936	2,352	0,13	18,86	22,1	65,5
0,02593	0,03936	2,352	0,1333	18,8	21,2	65,1
0,01881	0,03936	2,352	0,13	18,78	20,87	63,6
0,01303	0,03936	2,352	0,1317	18,75	20,63	63,0
0,004991	0,03936	2,352	0,135	18,8	20,0	60,0
38,6 mm pitch tube						
$K = 1{,}390 \times 10^{-7} e^{2898/Tm}$						
0,03105	0,05444	1,7145	0,2068	17,63	19,4	66,13
0,02406	0,05444	1,7145	0,2031	17;63	19,8	63,95
0,02154	0,05444	1,7145	0,2031	17,9	19,25	65,0
0,01690	0,05444	1,7145	0,1979	18,0	19,3	64,0
0,007947	0,05444	1,7145	0,2120	18,03	19,1	63,1

TABLE 2

Typical results for heat transfer determination

Velocity ($m\ s^{-1}$)	U ($W\ m^{-2}\ {}^{\circ}C^{-1}$)	N_{Nu}	N^1_{Re} $\times 10^{-3}$	N^1_{Pr} $\times 10^7$	K/Kf	De/F	n
27,4 mm pitch tube							
0,03179	16,66	2,122	2,259	2,549	0,1431	0,3226	0,855
0,02593	12,63	1,610	1,732	2,737	0,1419	0,3226	0,855
0,01881	11,05	1,411	1,062	3,105	0,1483	0,3226	0,855
0,01303	10,18	1,301	0,6641	3,392	0,1505	0,3226	0,855
0,004991	7,10	0,911	0,1728	4,565	0,1652	0,3226	0,855
38,6 mm pitch tube							
0,03105	18,77	3,322	2,496	4,874	0,0914	0,4462	0,85
0,02406	22,79	2,900	1,831	5,142	0,0916	0,4462	0,85
0,02154	14,42	2,555	1,475	5,461	0,0959	0,4462	0,85
0,01690	13,85	2,457	1,015	5,961	0,1003	0,4462	0,85
0,007947	12,44	2,208	0,3914	7,051	0,1035	0,4462	0,85



$(N_{Nu})_f = 5{,}12 \times 10^5 (N_{Re})^{0.0211} (N_{Pr})_f^{-0.481} (K/K_f)^{0.125} (De/F)^{2.74} (n)^{6.6}$

FIGURE 4 Comparison of measured and calculated Nusselt number.

diameter ratio and the flow behaviour index. The correlation coefficient was 0,935.

Power consumption

Sixty-five data points were obtained for the four cooling elements used in this study. The power number for each point was calculated from the relationship

$$N_{Po} = P/D^5.R^3.\rho \qquad (16)$$

The generalized Reynolds number and the ratio of the mean hydraulic diameter to the width of the fin was also calculated for each data point. Typical data points and results are listed in Table 3 and all the results are shown graphically in Fig. 5.

The following equation was obtained using regression analysis, the dimensionless numbers considered being the Reynolds number, diameter ratio and flow behaviour index.

$$N_{Po} = 3{,}89\ (N^1_{Re})^{-1,18}\ (De/F)^{0,917}\ (n)^{-9,11} \qquad (17)$$

The correlation coefficient of this equation was 0,984.

Discussion

In horizontal crystallizers in which the stirrer is completely immersed and in vertical crystallizers of conventional design rotation of the stirrer causes, to a certain extent, a rotation of the massecuite. Hence, the velocity of the massecuite relative to the cooling element cannot be measured by the product of the rotating speed and diameter of rotation, but

TABLE 3

Typical data and results for power determination

Velocity ($m\ s^{-1}$)	Massecuite Temp. (°C)	Dynamometer reading (kg.)	Power (W)	N_{Po}	N^1_{Re} $\times 10^{-3}$	De/F
$K = 1{,}0487 \times 10^{-10}.e^{9810/Tm}$					n = 0,8347	
52,8 mm pitch tube						
0,03572	60	17	6,260	3 259,8	7,2164	0,5862
0,03179	60	16	4,990	4 247,3	5,9625	0,5862
0,02309	60	14	3,170	7 092,9	4,0962	0,5862
0,01699	60	12	2,000	11 239,0	2,8644	0,5862
0,01081	60	9	0,953	20 824,0	1,6902	0,5862
0,00450	60	6	0,266	79 372,0	0,6108	0,5862
69,3 mm pitch tube						
0,03142	50	20	6,17	5 528,0	2,9212	0,7255
0,02573	50	18,4	4,77	7 731,6	2,3203	0,7255
0,02078	50	15,3	3,12	9 594,8	1,8093	0,7255
0,01589	50	14,0	2,19	15 065,0	1,3234	0,7255
0,01051	50	10,5	1,08	25 658,0	0,8178	0,7255

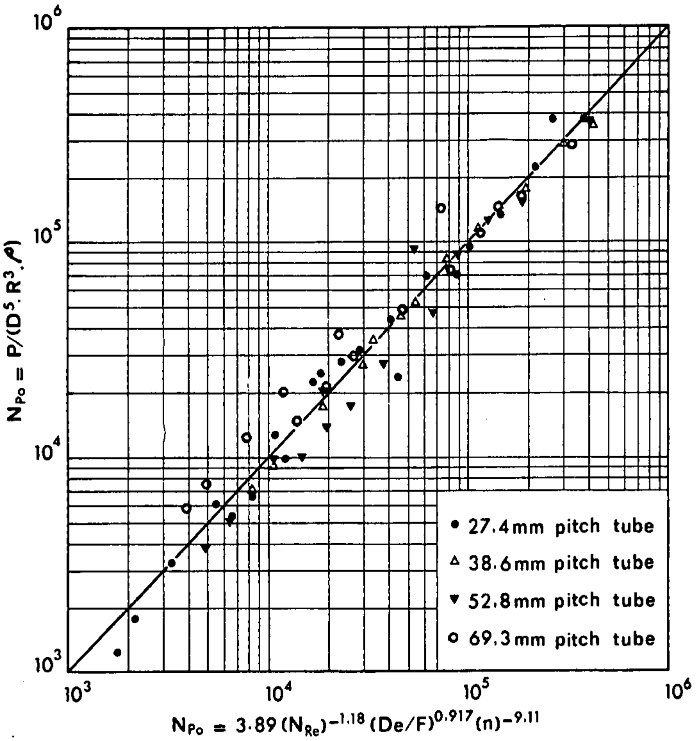


FIGURE 5 Comparison of measured and calculated Power number.

what is obtained instead is the apparent velocity. As this apparent velocity was used in evaluating the Reynolds number in equations (15) and (17), use of these equations is limited to the prediction of the overall heat transfer coefficient and power consumption of these crystallizers.

In horizontal crystallizers running half full the force of gravity tends to keep the level of the massecuite horizontal and as a result the velocity of the cooling elements relative to the massecuite is increased. This produces an increased power consumption and probably an increase in the overall heat transfer coefficient. The increased power demand of half empty crystallizers has been observed previously in Mauritius (le Guen[6]).

The power predicted by equation (17) is that which is required for stirring the massecuite. The total power required by a cystallizer will be greater than that predicted by equation (17); Le Guen[6] reporting mechanical losses of 1,2 kw for a Fletcher-Blanchard crystallizer of 30 m³ capacity.

Practical applications

The heat transfer rate in a crystallizer is obtained by calculating the mean value of the overall heat transfer coefficient based on the individual heat transfer coefficient of each tube taking into account its distance from the axis of the stirrer. Similarly the power demand is the sum of the power required by each tube.

In Figures 6 and 7 are shown the overall heat transfer coefficient and power requirements of a crystallizer having the dimensions shown in Fig. 8. As can be observed, when massecuites are cooled down to about 40°C, the overall heat transfer coefficient is only slightly affected by the speed of

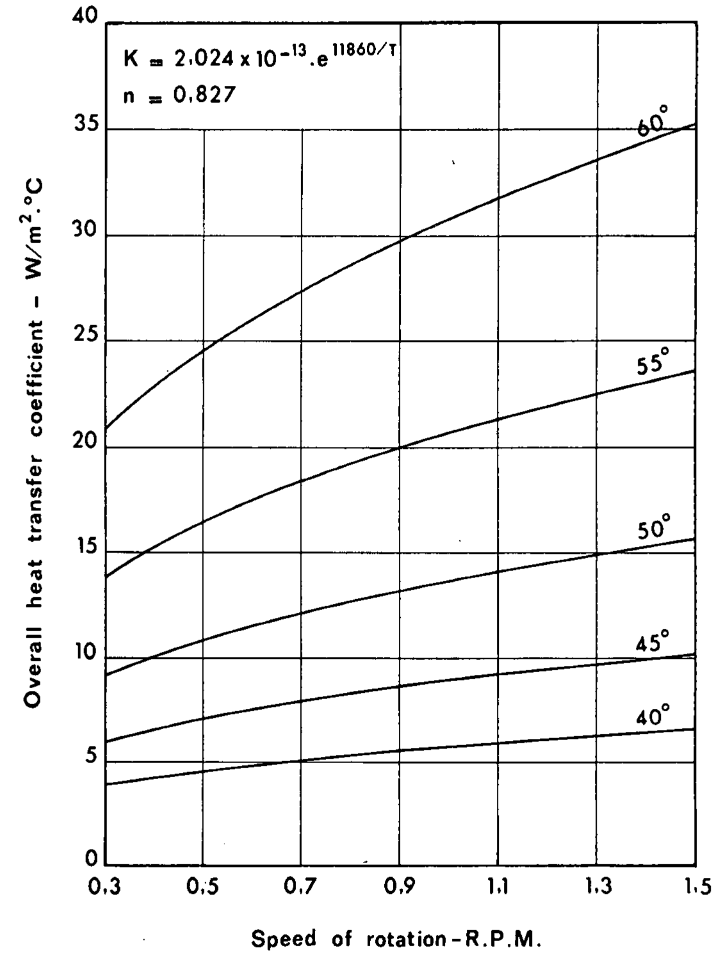


FIGURE 6 Overall heat transfer coefficient of crystallizer as a function of speed of rotation and massecuite temperature.

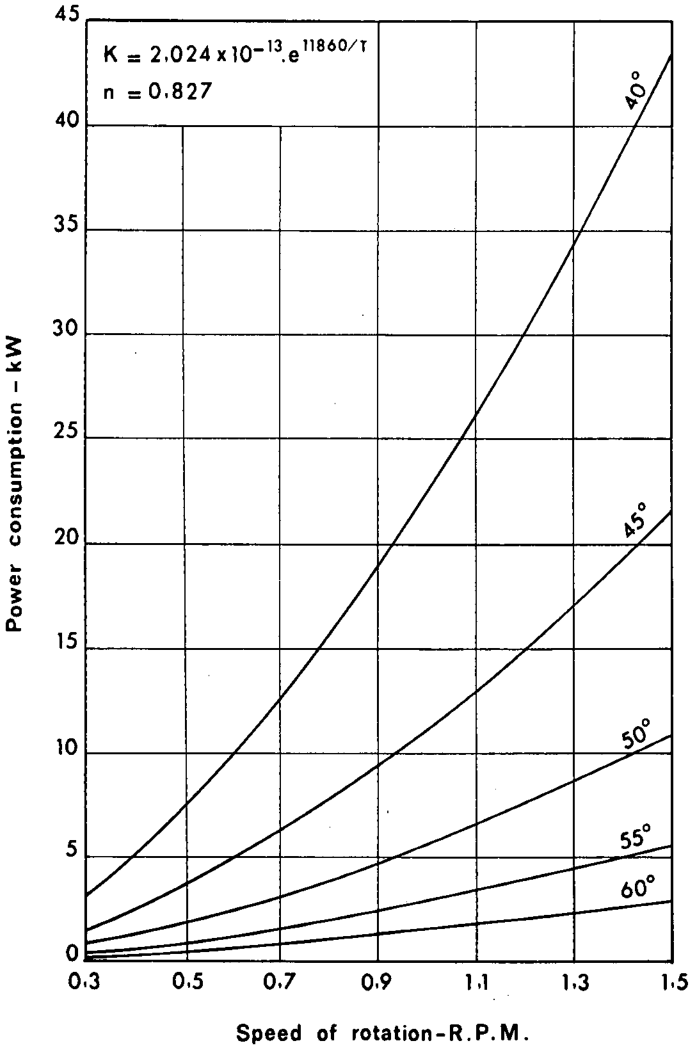


FIGURE 7 Power consumption of crystallizer as a function of speed of rotation and massecuite temperature.

rotation, but increasing the speed causes a rapid increase in the power demand.

However when applying these equations for the calculation of crystallizers, it must be remembered that they were derived from experiments done on a pilot plant, and that until measurements are made on full size units they should be used with caution.

Acknowledgements

Thanks are due to the staff of the SMRI workshop who constructed the plant, to Mr G. N. Allan who designed the dynamometer and gave valuable advice, to Mr J. FitzGerald of Smithtech (Pty.) Limited who provided the working drawings of the crystallizer and to Mr D. Hoyer who did the power measurements. Their assistance is gratefully acknowledged.

Nomenclature

The symbols used in the text are listed below:

A	=	total heat transfer area	m^2
Bx	=	brix of massecuite	
Cp	=	heat capacity	$J.kg^{-1}.°C^{-1}$
D	=	diameter of rotation	m
De	=	mean hydraulic diameter	m
F	=	width of fin	m
h	=	film heat transfer coefficient	$W.m^{-2}.°C^{-1}$
K	=	power law consistency index	$kg.m^{-1}.s^{n-2}$
k	=	thermal conductivity	$W.m^{-1}.°C^{-1}$
n	=	power law flow behaviour index	
N_{Nu}	=	Nusselt number	
N_{Po}	=	power number	
N_{Pr}	=	Prandtl number	
N^1_{Pr}	=	generalized Prandtl number	
N_{Re}	=	Reynolds number	
N^1_{Re}	=	generalized Reynolds number	
P	=	power	W
R	=	rotational speed	s^{-1}
Sr	=	shear rate	s^{-1}
T	=	absolute temperature	°K
t	=	temperature	°C
Δt_{lm}	=	logarithmic mean temperature difference	°C
U	=	overall heat transfer coefficient	$W.m^{-2}.°C^{-1}$
V	=	velocity	$m.s^{-1}$
W	=	mass rate of flow	$kg.s^{-1}$

Greek:

μ	=	viscosity	$kg.m^{-1}.s^{-1}$
ρ	=	density	$kg.m^{-3}$
τ	=	shear stress	$kg.m^{-1}.s^{-2}$

Subscripts

f	=	measured at average film temperature
i	=	inlet conditions
m	=	massecuite
o	=	outlet conditions
s	=	surface of tube
w	=	cooling water

REFERENCES

1. Gromkovskii, A. I. (1967). "Determination of parameters of final product massecuite", Sakhar Prom., **41**, 30-35.
2. Gulyi, I. S. (1965). "Intensity of heat exchange during the cooling of massecuite in crystallizers", Sakhar Prom., **39**, 107-114.
3. Honig, P. (1952). "The technology of crystallizers", Sugar J. **15** (4) 16-22; **16** (6) 17-23; **18** (2) 38-44.
4. Honig, P. (1953). Principles of sugar technology, Elsevier, Amsterdam.
5. Hugot, F. (1972). Handbook of sugar cane technology, 2nd edition, Elsevier, Amsterdam.
6. Le Guen, F. and De R. dè Saint Antoine, J. D. (1963). "Notes on the power consumption of C-massecuite crystallizers". MSIRI Annual Report, 127-131.
7. Wilkinson, W. L. (1960). Non-Newtonian fluids: Fluid mechanics, mixing and heat transfer. Pergamon Press, London.

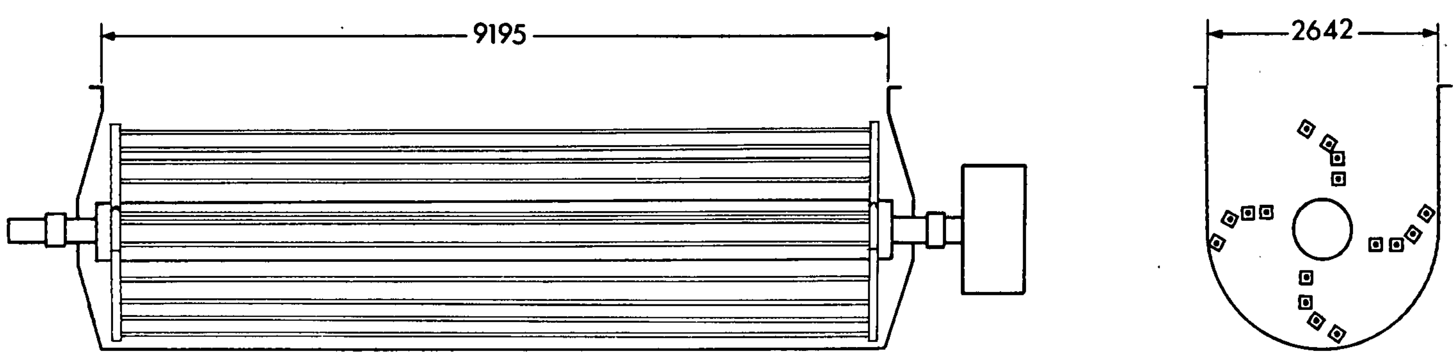


FIGURE 8 Dimensions of crystallizer used to illustrate calculation of overall heat transfer coefficient and power consumption (Smithtech).