USING FINITE ELEMENTS ANALYSIS AND DESIGN THEORY AS TOOLS TO DESIGN LARGE DIAMETER SHAFTS

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Abstract

One of the problems on designing a large diameter shaft with changes in diameter is to obtain the correct Stress Concentration Factors (SCF) at the change of diameter. Detail curves are available for shafts where the fillet radii are in general larger than 2.5% of the smallest diameter on the shaft. These curves are also only for a single radius fillets. On many of the large diameter shafts one is limited on maximum single fillet radius, or one needs to use more than one radius to reduce the SCF to an acceptable value at the step where the shaft changes diameter. Both these cases fall outside the standard curves available for SCF. The advancement in computing power and in the finite element analysis programs allows the designer to use finite element analysis as a design tool to determine these SCF. These factors are essential in determining the static strength and fatigue life of a specific shaft.

This paper shows how the SCF are calculated and how these factors can be used with design theory to calculate the static strength and fatigue life at a point on a large diameter shaft.

Keywords: finite element analysis, design, large diameter shafts

Introduction

Designing large diameter shafts has always been an intensive process to ensure that all factors influencing the design have been considered for an optimal design. It is also important that the shaft is designed for the correct criteria, static load or infinite fatigue life or finite fatigue life. Theory exists to design shafts for these three criteria as discussed in Shigley (1986) and Benham and Crawford (1987). The important areas of the shaft that need to be checked are those where the shaft changes in geometry. There is a stress concentration in these areas due to the change in geometry. Stress concentration factors exist for shaft with only a single fillet radius where the shaft changes in diameter Shigley (1986). It is sometimes required to use two fillet radii to reduce the stress concentration to an acceptable level.

This paper looks at a large diameter shaft with an increase in diameter. Two fillet radii have been used at the step change, to reduce the stress concentration. Finite Element Analysis (FEA) has been used to calculate the Stress Concentration Factor (SCF) for both bending and torsional stresses. Design theory can then be used with these SCFs to design the shaft according to the desired criteria; an infinite fatigue life has been selected for the example.

General

In designing a shaft one needs to consider the loads on the shaft. There are normally two loads on horizontal shafts, namely a bending load and a torsional load; sometimes an axial load might exist but this is excluded in the paper. When the two loads, bending and torsion, are considered one must ensure that it is clear which load is of a cyclic nature and which is of a constant nature. Figure 1 shows a diagrammatic layout of a shaft with a torque, T, and a force, F, applied to it at two points. This loading results in a total reaction torque, Tr, and two reaction forces, R. The applied loading stays constant, which results in a constant bending and torsional loading on the shaft. Rotation of the shaft causes alternating bending stress.

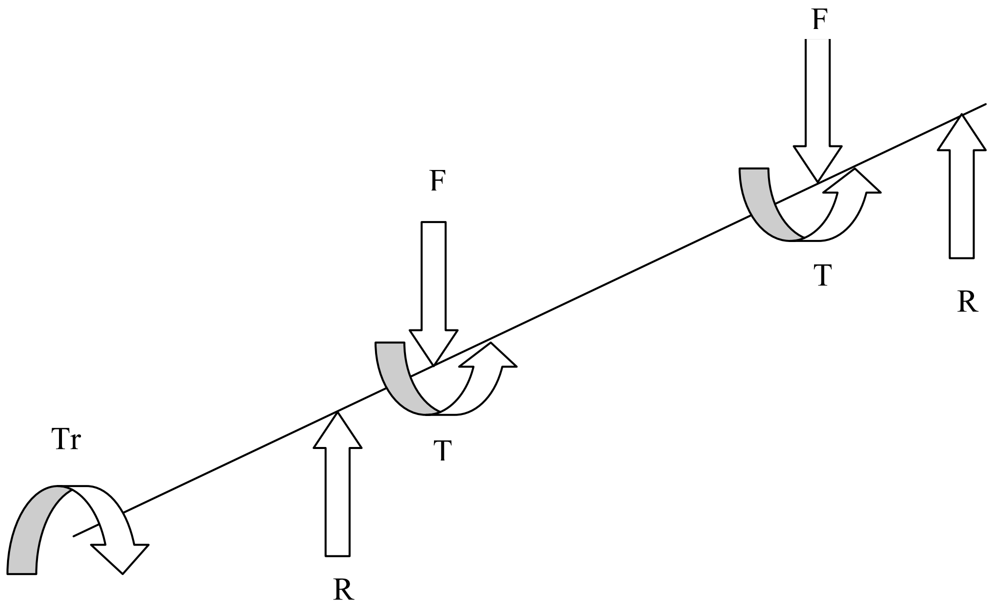

Figure 1. Diagramatic view of shaft with a bending and torsional load.

Determining stress concentration values for shaft

A section of the shaft has been modelled in a FEA program to determine the SCFs for the shaft. Two analyses were done on the shaft, one for the SCF due to bending and one due to torsion on the shaft. The section of the shaft used in the FEA is shown in Figure 2.

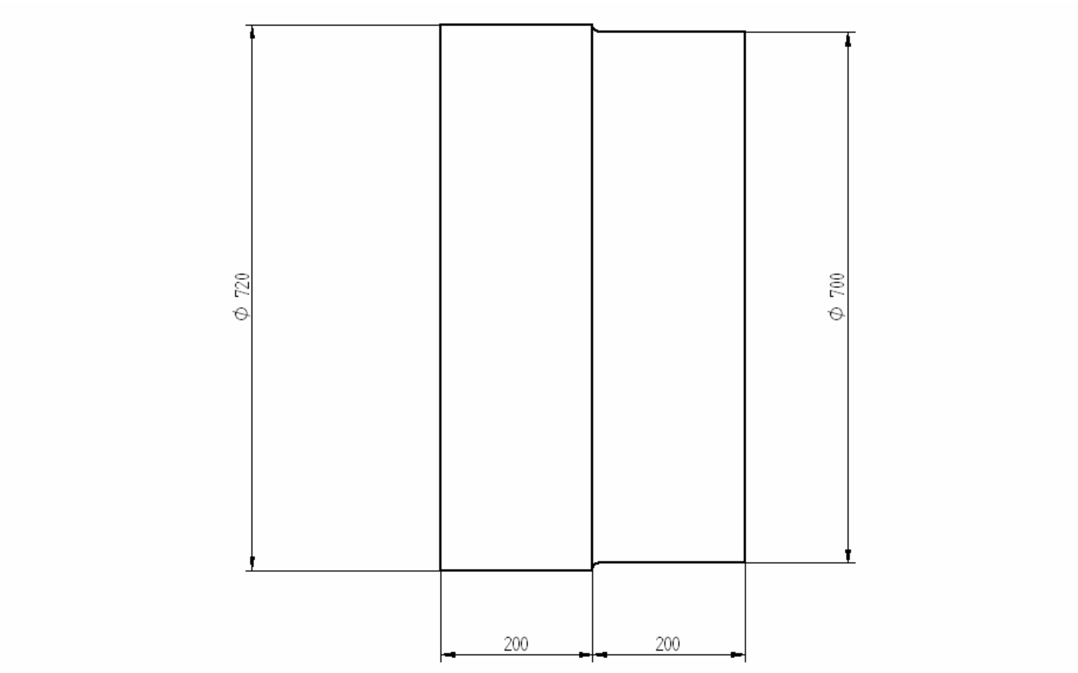

Figure 2. Section of shaft.

Figure 3 show the detail of the fillet radii used for the example. The 20 mm radius is tangential to both the 700 mm diameter section of the shaft and the 5 mm radius. The 5 mm radius ends tangential on the vertical section with a 4 mm vertical shoulder. The alternative to this arrangement would be to use only a 6 mm fillet radius, which will also give a 4 mm vertical shoulder on the shaft. The SCF from bending is approximately 2.2 and the SCF from torsion is approximately 1.8, as illustrated in Figure A-23-9 and Figure A-23-8 respectively in Shigley (1986). These SCFs are compared to the SCFs determined with FEA in Table 1 later in the paper.

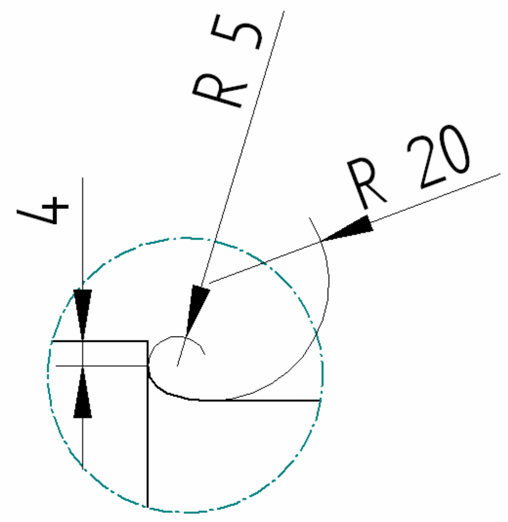

Figure 3. Detail of fillet radii.

The geometry was meshed with parabolic tetrahedral elements as shown in Figure 4. A finer mesh has been used in the area where the shaft changes in section. Two different loads were applied to the model, first a bending load and secondly a torsional load. The bending load was a bending moment of 1 Nm and the torsional load was torsion of 1 Nm.

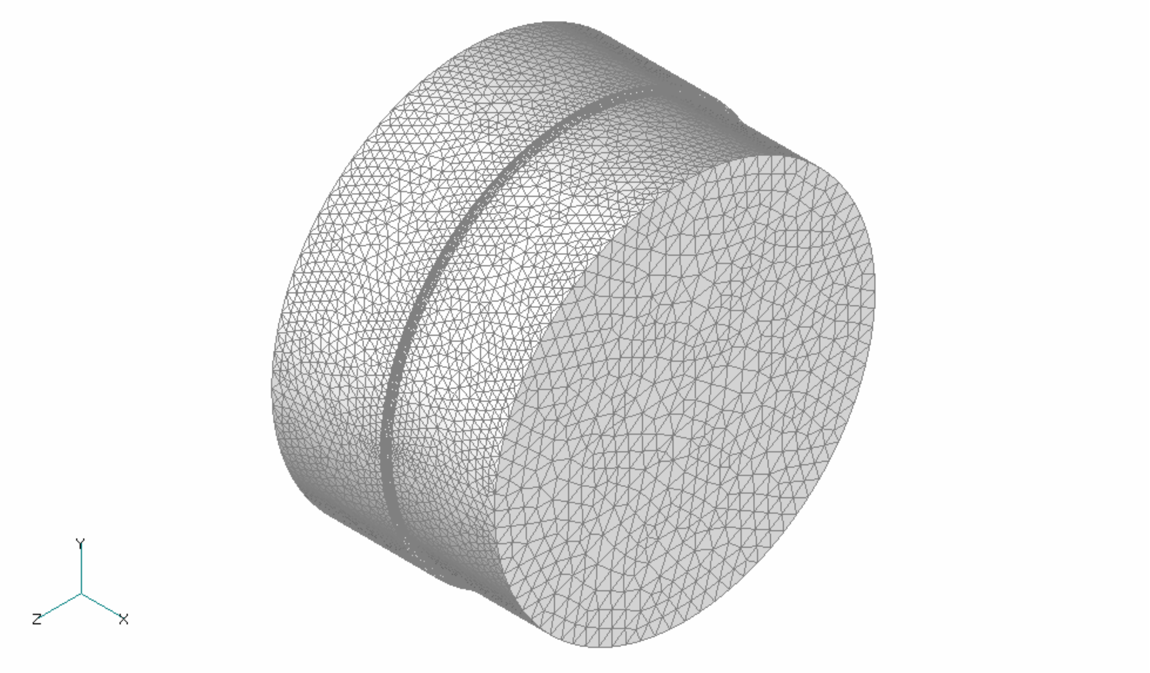

Figure 4. Mesh of shaft section.

FEA for bending load

Figure 5 shows the shaft model with bending load and constraints applied to it. A moment of 1 Nm was applied around the Y axis. This moment is connected to the model through a rigid element, which is not shown in this figure for clarity. The model with the rigid element is shown in Figure 6. As seen the rigid element connects the node where the load has been applied to smaller diameter face of the shaft. This specific rigid element allows free movement of all nodes on the face in the Y and Z directions as well as free rotation around the X axis. Movement in the X direction and the rotation around Y and Z axes are determined by the unit moment applied to the model.

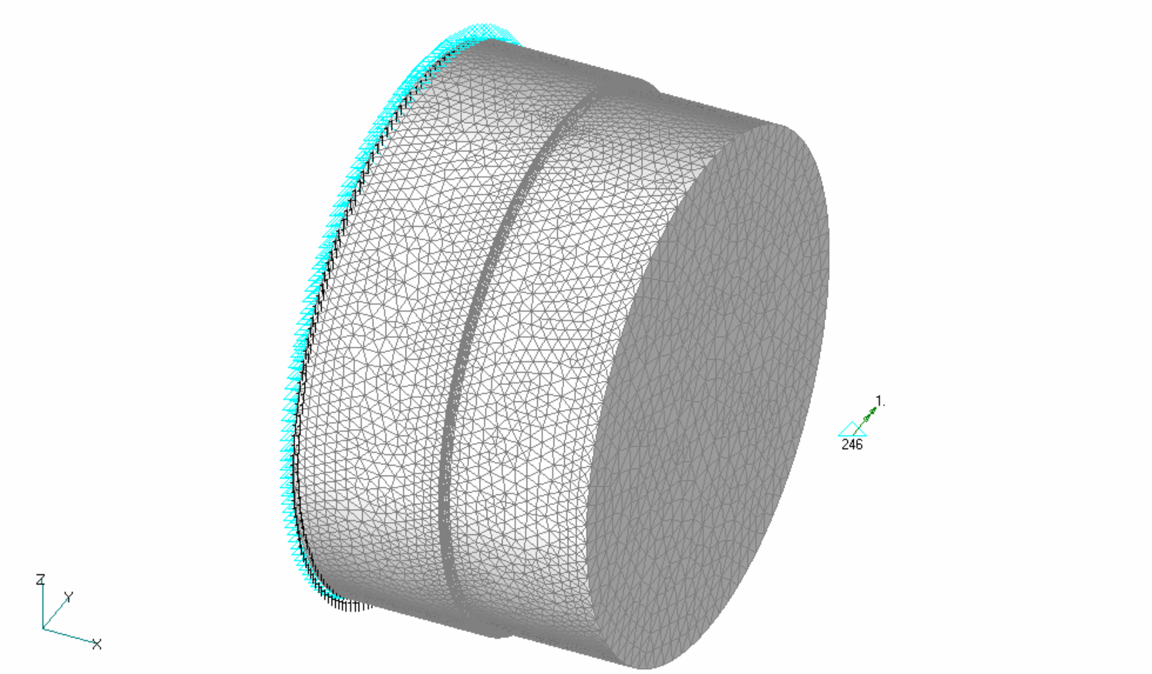

Figure 5. Shaft with unit bending moment and constraints (rigid element excluded).

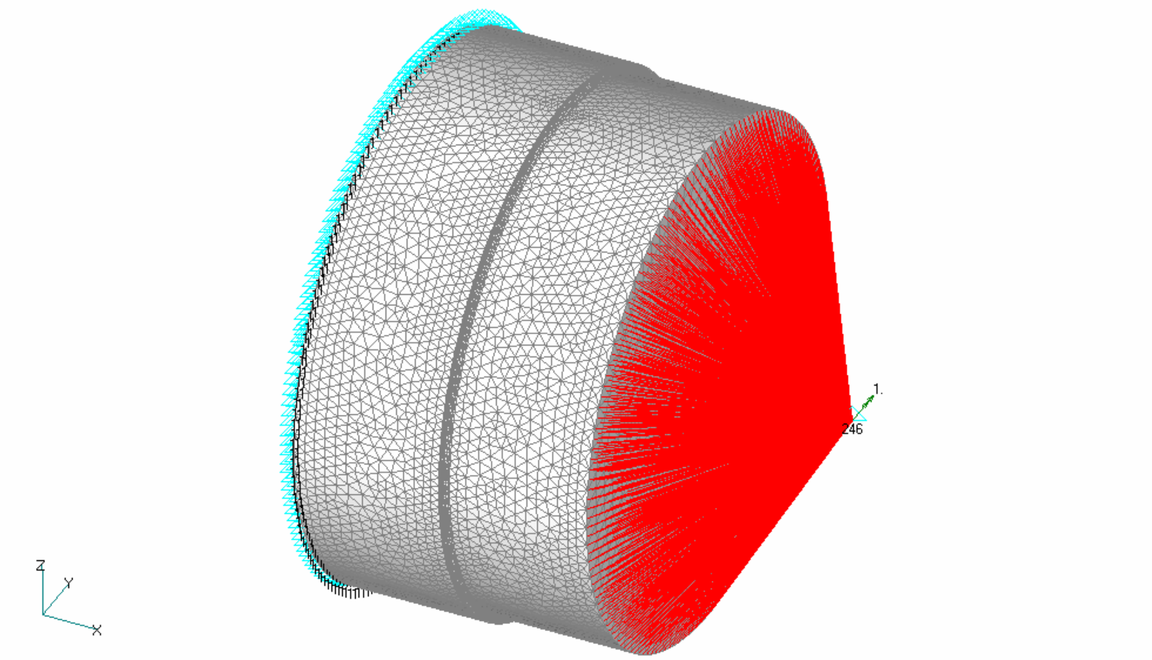

Figure 6. Shaft with unit bending moment and constraints (rigid element included).

The face of the larger diameter of the model is constrained in the X direction. The nodes on this face are free to move in the Y and Z directions as well as free to rotate around all three axes.

The total Von Mises stress of 52.14 Pa was calculated by the program and is shown in Figure 7. Von Mises stress is a stress that computes a single value from the stress condition on an element. The value can then be compared with the yield stress of the material of that element, as discussed in Benham and Crawford (1987).

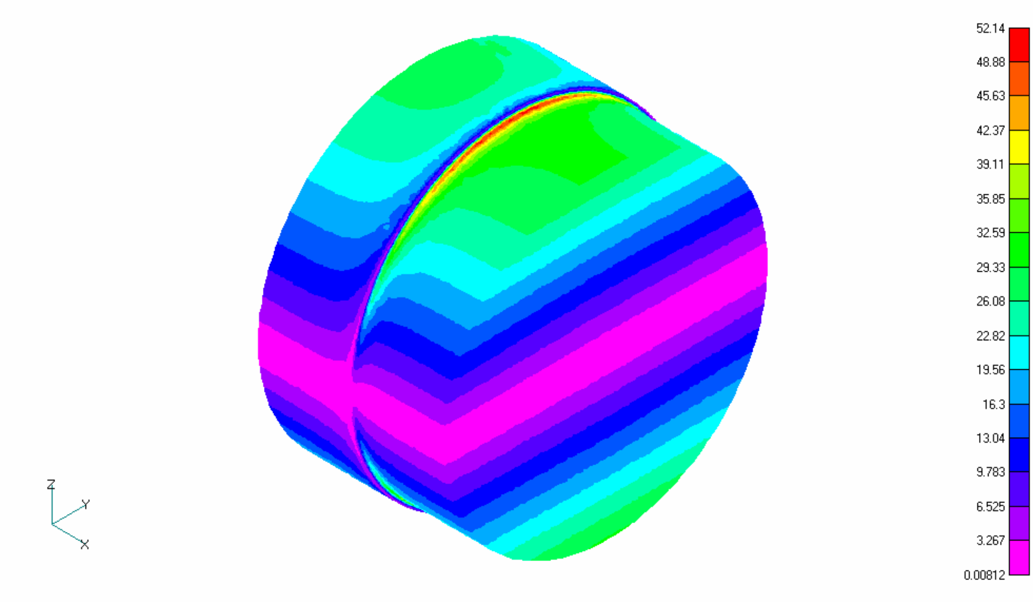

Figure 7. Von Mises stress due to a unit bending moment.

FEA for torsional load

Figure 8 shows the shaft model with torsional load and constraints applied to it. A torsional moment of 1 Nm in total was applied to the model around the X axis. This torsional moment is applied to the curve on the edge of the face of the smaller diameter section in Nm/length. The model with the rigid elements is shown in Figure 9. As seen the rigid elements connects the nodes where the constraints have been applied to the two faces of the shaft. Movement in all directions and rotation around all axes of the nodes on these two faces are determined by the rigid elements and by the two constraints applied.

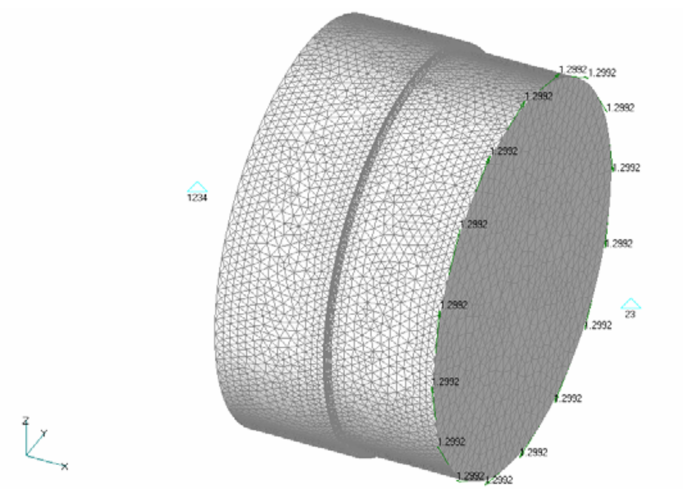

Figure 8. Shaft with unit torsional moment and constraints (rigid elements excluded).

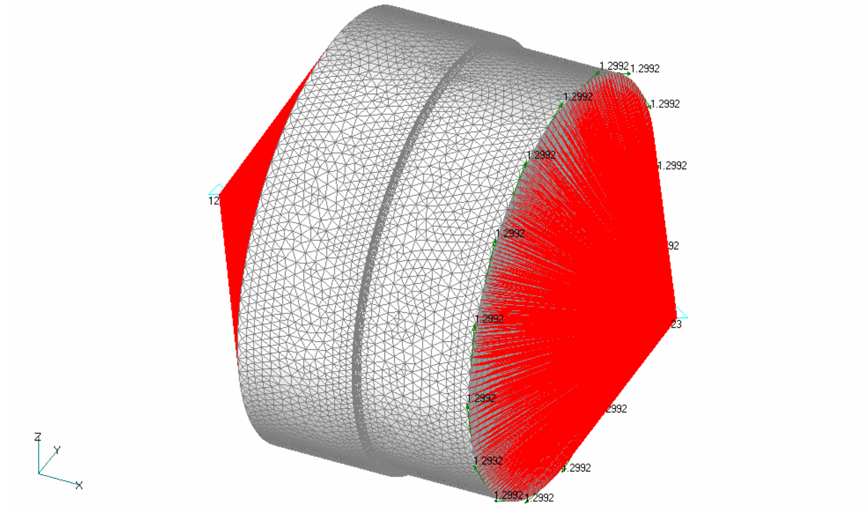

Figure 9. Shaft with unit torsional moment and constraints (rigid elements included).

Total Von Mises stress of 41.21 Pa was calculated by the program and is shown in Figure 10.

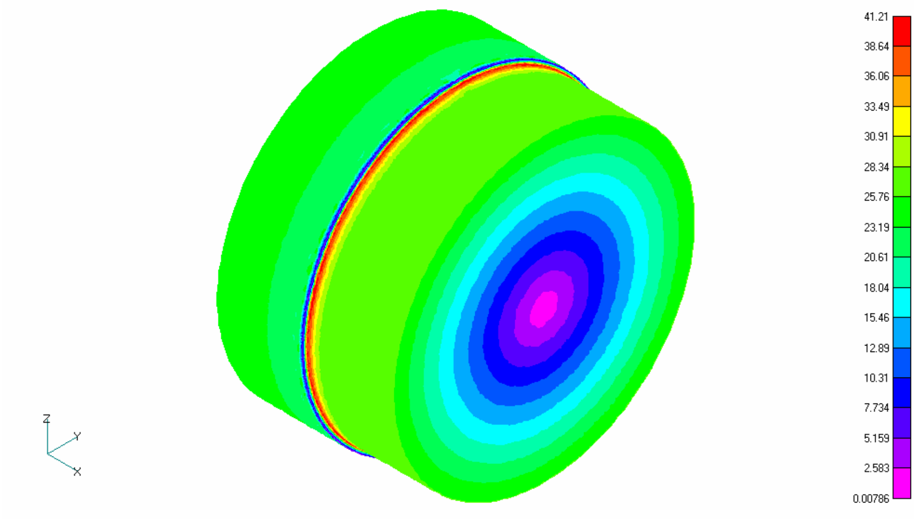

Figure 10. Von Mises stress due to a unit torsional moment.

Analytical methods

The analytical values of the Von Mises stresses for both the bending and torsional load can be calculated using equations and methods obtainable in Shigley (1986) and Benham and Crawford (1987). These calculations have been done and are shown in Appendix A. The SCFs can now be calculated by dividing the FEA Von Mises stresses with the analytical values. A summary of the stresses and the SCFs is shown in Table 1.

Table 1. Summary of FEA and analytical Von Mises stresses and SCFs.

	FEA (Pa)	Analytical (Pa)	SCF (FEA)	SCF (Shigley)
Von Mises stress due to bending	52.14	29.70	1.7555 ≈ 1.76 SCFB = 1.76	SCFB = 2.2
Von Mises stress due to torsion	41.21	25.72	1.6023 ≈ 1.61 SCFT =1.61	SCFT = 1.8

Note that the SCF values in Table 1 have been rounded up; this is to ensure that the rounding error is made to a higher value, which will ensure a safer design of the shaft. The SCFs as calculated with FEA are lower than those obtained from Shigley (1986): this will result in a lower stress at the geometry change of the shaft for the same change in shaft diameter.

These SCFs, with the applied loads, can now be used to design the shaft according to the desired criteria. Having the SCFs available also allows the designer to use different loads on the shaft. The static load design criteria were selected, as an example to show how the SCFs are used.

Example

Bending and torsional SCFs were calculated for the shaft with a change in geometry as shown in Figures 2 and 3. These SCFs (Table 1) together with a selected loading condition (bending and torsional load) on the shaft can now be used to calculate the stresses due to the loading. It is assumed that the Bending Moment at the fillet radii is 3500 kNm and that a constant Torque of 4000 kNm is applied to the shaft. The bending stress and the torsional shear stress can now be used to calculate the static Von Mises stress. This Von Mises stress value can then be compared with the Yield Stress of the shaft material. If the Von Mises stress is smaller than Yield Stress the shaft should be safe against static yielding.

Shaft with selected bending moment and torsional load:

ID	Internal diameter of shaft	0	mm
OD	Outer diameter of shaft	700	mm
M	Applied bending moment	3500000	N.m
T	Applied torque	4000000	N.m
SCF Bend		1.5	
SCF Torque		1.5	
I	Second moment of area	0.01179	m^4
J	Polar area moment of inertia	0.02357	m^4
σ_B	Bending stress	1E+08	Pa
τ	Torsional shear stress	5.9E+07	Pa
σ_B	Bending stress	1.6E+08	
τ	Torsional shear stress	8.9E+07	
σ_1	First principle stress	2E+08	Pa
σ_2	Second principle stress	-4E+07	Pa
σ_{VM}	Von Mises stress	219.358	MPa

A high strength material will normally be used for an application like this. Typical yield stress for such a material will be above 450 MPa and the typical ultimate tensile stress will be above 680 MPa; it is assumed that the two values to be the yield and ultimate tensile stress for the shaft. The calculated Von Mises stress of 220 MPa (rounded up) is less than half the value of the selected yield strength, which indicates that the shaft should be safe against static failure with a factor of safety of 2.045 against static yielding.

Conclusion

It is clear from the example that FEA is a handy tool to optimise the geometry on a shaft. Various options of radii sizes and orientations can be used in the FEA to reduce the stress concentrations in the area where the shaft changes geometry. This allows the designer to optimise the shaft design with the selection of material and shaft diameter. These two factors are the two biggest contributors to the cost of a large diameter shaft.

The example calculation showed how the static Von Mises stress has been calculated. Other calculations can be done to determine either a finite or infinite fatigue life of the shaft. The same SCFs would be used in those calculations. Examples of these calculations can be found in (Shigley, 1986).

REFERENCES

Benham PP and Crawford RJ (1987). *Mechanics of Engineering Materials.* Longmans Scientific and Technical, Harlow. 626 pp.

Shigley JE (1986). *Mechanical Engineering Design*. First Metric Edition, McGraw-Hill Book Company, New York, USA. 699 pp.

APPENDIX A

Shaft design calculation for Bending Moment SCF

OD	Outer diameter of shaft	700	mm
M	Applied bending moment	1	N.m
T	Applied torque	0	N.m
I	Area moment of inertia	0.01179	m^4
J	Polar area moment of inertia	0.02357	m^4
σ_B	Bending stress	29.6965	Pa
τ	Torsional shear stress	0	Pa
σ_1	First principle stress	29.6965	Pa
σ_2	Second principle stress	0	Pa
σ_{VM}	Von Mises stress	3E-05	MPa
		29.6965	Pa

Shaft design calculation for Torsional SCF

OD	Outer diameter of shaft	700	mm
M	Applied bending moment	0	N.m
T	Applied torque	1	N.m
I	Area moment of inertia	0.01179	m^4
J	Polar area moment of inertia	0.02357	m^4
σ_B	Bending stress	0	Pa
τ	Torsional shear stress	14.8483	Pa
σ_1	First principle stress	14.8483	Pa
σ_2	Second principle stress	-14.848	Pa
σ_{VM}	Von Mises stress	2.6E-05	MPa
		25.718	Pa