

ANALYSIS OF CRYSTAL RESIDENCE TIME DISTRIBUTION AND SIZE DISTRIBUTION IN CONTINUOUS BOILING VACUUM PANS

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Abstract

The results of residence time distribution tests on a number of continuous vacuum pans in South Africa are presented. A model incorporating a number of well-mixed tanks-in-series is developed to characterise the massecuite flow and facilitate comparisons. The effect of a spread in residence times on sugar crystal size distribution is investigated and results presented.

Introduction

The development of the chemical processing industries over the years has been characterised by trends towards bigger processing units to achieve economies of scale, and from batch to continuous equipment to achieve economies in capital, operating costs and improvements in product quality. Batch equipment is well suited to produce small amounts of material, or to produce many different products from one piece of equipment. On the other hand, for the treatment of materials in large amounts, the continuous process is nearly always found to be more economical.

The history of sugar processing has shown the same two trends. Vacuum pan boiling is the last major unit operation in the sugar mill which is still generally carried out batch-wise. The change to continuous boiling systems has considerable advantages, which have been outlined elsewhere.⁹ However, the transition to continuous pan boiling has been retarded by a few major problems. One of these has been the realisation of a product which has a much wider range of crystal sizes than is obtainable in the batch system.

The conversion of certain processes from batch to continuous mode has not always been successful, as evidenced by some of the problems experienced with continuous clarifiers and crystallisers. The main reason for these problems is that insufficient consideration has been given to the flow system. If deviations from plug flow are very large, as has been found in some cases, then the batch system can be more economical than the continuous system. Similarly with continuous vacuum pans, attention to the massecuite flow system is of fundamental importance. In a batch pan, the crystal residence time is constant (assuming that control is such that no nucleation or dissolution of crystals occurs) and all crystals remain in the system for the same length of time. In continuous pan boiling, the aim is still to maintain a constant crystal retention time, but this is only possible in the case of a true plug flow system.

For this reason, the massecuite flow system was thoroughly investigated in developing the new Tongaat-Hulett continuous pan design. Tracer techniques have been used in the evaluation of different continuous pans as a means of characterising massecuite flow patterns and crystal residence times.

A 'C' massecuite batch pan at Mount Edgecombe was converted into a 6-compartment continuous pan in which the first continuous pan tracer test was carried out. This was followed by tracer tests undertaken by the Sugar Milling Research Institute (SMRI) on Fives-Cail Babcock (FCB) continuous pans at Gledhow and Tongaat.^{3,7}

A further tracer test was undertaken on a modified 'A' massecuite batch pan at Amatikulu, prior to the design and installation of the new Tongaat-Hulett Sugar (T-H.S) continuous pan at Maidstone. Tracer tests were undertaken on this pan as

well, while boiling all three grades of massecuite, and some results have been obtained from the T-H.S continuous pans at the new Felixton mill during the first season of operation.

To gain some quantitative information on the flow systems, a modified model, based on the tanks-in-series model, has been developed and applied to these tracer tests. This facilitates comparison of flow systems in different pans.

The reason for the preoccupation with flow patterns and residence times is to reach the objective of producing crystals of a uniform size. This is particularly important with 'A' sugar which has to meet specifications on grain size and fines, but is equally desirable in 'B' and 'C' boilings to facilitate the centrifuging operation. Thus, in addition to tracer tests, a large number of grain size analyses have been undertaken, mainly on the continuous 'A' pans.

Development of Mathematical Models

Residence time distribution

A tanks-in-series model is useful for determining the approach of the flow system to plug flow. With this model, a continuous flow system is visualised as a series of n equal-sized, well-mixed tanks-in-series. The equation for this model is:⁵⁻

$$E(t) = \frac{n^n (t/\tau)^{n-1}}{(n-1)!} \exp(-nt/\tau) \dots \dots \dots (1)$$

where $E = C\tau/C_0$

$C =$ tracer concentration

$n =$ No. of tanks

$\tau =$ mean residence time

$C_0 =$ concentration scaling factor

$t =$ time

A system with complete back-mixing would be represented by 1 tank, whereas an ideal plug flow system is simulated by an infinite number of tanks-in-series. Most flow systems fall somewhere between the 2 extremes. In this model, the flow rate through the system is assumed to be constant. This unfortunately does not apply to the continuous pan system, since there is progressive feed of syrup or molasses along the length of the pan. A modification has therefore, been made to this model, to allow for feed addition.

In most cases, the feed will not necessarily be uniform along the pan, due to changes in massecuite crystal content and brix. However, it has been necessary to assume a uniform addition of feed to each tank, in order to keep the mathematics tractable. This model can be formulated either via a stochastic approach, or by a deterministic approach. The model obtained from the stochastic approach attempts to account directly for the random nature of the flow system. The deterministic flow system models account for the unpredictability by incorporating parameters which can be chosen so that the model output matches the experimental output. Both approaches yield the same answer and the stochastic approach is shown in Appendix 1. The final model equation is as follows:—

$$E(t) = \prod_{i=1}^n ((v_n - (n-i)x)/V) \cdot \sum_{j=1}^n \frac{\exp(-(v_n - (n-j))t/V)}{\prod_{\substack{k=1 \\ k \neq j}}^n ((k-j)x/V)} \quad (2)$$

The nomenclature used is listed at the end of the paper.

A non-linear regression routine was used to fit this equation to the measured data. The procedure involved varying values of the model parameters until the best fit between data and the model was obtained, based on the minimum sum of squared errors. A correlation coefficient was then calculated.

In the first series of tracer tests, neither the seed feed to the pan, nor the massecuite production rate was measured. In this case, the fitted parameters were the number of tanks-in-series, the initial concentration C_0 and the seed and massecuite flow rates. In later tests where these flow rates were measured, the parameters fitted by the model were the number of tanks-in-series, the initial concentration C_0 and the massecuite volume in the pan. In addition, the tracer recovery was calculated.

The fitted volumes might be slightly different from the nominal volumes of the pans, firstly because of the difficulty in assessing the average massecuite level in a vigorously boiling pan, and secondly due to the effect of boiling on the apparent massecuite volume. Nevertheless, the fitted volume should approach reasonably closely the expected volume, and the tracer recovery should be 100% or close to it, otherwise the results must be doubtful.

Crystal size distribution

The size distribution of crystals produced in a batch pan is affected by the seed introduced into the pan (ie the size distribution of the starting material) and various size dispersion effects. Wright & White¹¹ list a number of possible mechanisms which can contribute to a spreading of the crystal size distribution during growth. These are:

- (a) poor mixing in the pan,
- (b) nucleation, crystal break-up, attrition and conglomeration,
- (c) crystal residence time variations,
- (d) size-dependent growth rates,
- (e) inherent variations in crystals.

Item (c) is of importance only in continuous systems, since in batch pans all crystals have the same residence time (provided secondary nucleation or false grain is avoided). Austmeyer¹ reported that large crystals grow faster than small crystals, but Wright & White¹¹ concluded that size-dependent effects are important in some crystallisation systems but not in the case of sucrose. Then, if control of the process is good, (b) is probably not important and so (a) and (e) are expected to be the major factors.

The effect of the residence time distribution on the crystal size distribution can be predicted using the tanks-in-series model. From the model, the expected mean value is τ and the variance τ^2/n . If the effective growth rate of the crystals is w , then the mean crystal size will increase as follows:

$$\mu_r = \mu_i + w\tau \dots \dots \dots (3)$$

where μ_r is the expected mean value of the distribution of crystals in the final massecuite and μ_i is the expected mean value of the distribution of crystals in the seed fed into the pan.

Also, since inlet crystal size and residence time are mutually independent random variables, the variance of the product is the sum of the variances of the seed and time distribution functions:

$$\sigma_r^2 = \sigma_i^2 + w^2 \tau^2/n \dots \dots \dots (4)$$

Equation (4) can then be used to predict the variance of crystal sizes in the massecuite, based on the distribution of inlet crystal sizes and residence times.

Wright & White¹¹ measured the overall size dispersion in sugar boiling systems in the laboratory. They found that the

variance of the size dispersion effect σ_d^2 in batch boilings where crystal residence times are constant could be expressed as:-

$$\sigma_d^2 = p\Delta L \dots \dots \dots (5)$$

where ΔL is the increase in mean crystal size and p is a proportionality factor. Since ΔL is given by $w\tau$, this can be written as:-

$$\sigma_d^2 = pw\tau \dots \dots \dots (6)$$

These effects will contribute to the variance, but not the mean of the product crystal size distribution. Introducing this into equation (4) gives:-

$$\sigma_r^2 = \sigma_i^2 + w^2 \tau^2/n + pw\tau \dots \dots \dots (7)$$

These quantities are all expressed in terms of a number distribution, not a mass distribution. In practice it has been found that sugar crystals in massecuites can be represented by a log-normal distribution.² The relationships between means and variances on a number and mass basis are given in appendix 2. They are used to convert mass distribution data obtained from sieve analyses into number distributions and vice versa.

Experimental details

Tracer testing

Lithium chloride was used as the tracer and was prepared according to the method given by Love.⁶ The quantity of lithium injected was calculated to give a maximum concentration of approximately 35 ppm in the massecuite.

In the most recent tests done on the continuous pans at Felixton, the tracer was injected directly into the first compartment. It was thought that this would minimise any diffusion of the tracer slug which is a change from earlier work where the seed pump was the usual point of injection.

Sampling rates were based on the pan volume and expected massecuite flow rate. They were calculated to give maximum coverage during the tracer peak emergence and a less frequent but complete coverage of the tracer tail, which is necessary for good model fitting.

During the tests at Mount Edgecombe and Felixton, intermediate compartments were sampled as was the final compartment, in order to give a more complete picture of the flow pattern in the pan. Flow rates were measured during the test periods, the most important being the rate of final massecuite output, molasses feed and seed feed. All of these were obtained by tank/crystalliser level variations over measured time periods. As far as possible, conditions in the pans were kept constant for the test period, because fluctuations in flow rates, absolute pressure, or calandria pressure have adverse effects on the flow characteristics of the pan.

The samples for lithium analysis were prepared in the Tongaat-Hulett Research and Development laboratories at Mount Edgecombe and the actual analysis was carried out using flame emission spectrophotometry at the SMRI.

Grain size analysis

Several sets of samples comprising seed, intermediate compartment massecuite and final massecuite were taken during each test. In the case of 'A' pans, it was necessary to centrifuge these samples immediately to prevent further crystallisation and to obtain a suitable crystal sample. This was done in a Christ laboratory centrifuge fitted with a perforated basket and a screen with 0,5 mm x 2,5 mm slots. After washing with alcohol and ether, the sugar was then screened using 14, 16, 20, 28, 32, 42 and 65 mesh screens and MA and CV were calculated from the weights of the various fractions.

Description of pans tested

Tracer tests were carried out on 'C' massecuite FCB pans at Maidstone (64 m³) and Gledhow (104 m³). These pans are de-

scribed by Jullienne & Munsammy.³ Batch pans converted into continuous pans by inserting radial baffles to provide 6 compartments were tested at Mount Edgecombe and Amatikulu.⁹ The new T-H.S pan boiling 'A' massecuite at Maidstone is described by Kruger⁴ while the T-H.S continuous pans boiling 'A', 'B' and 'C' massecuite at Felixton are described by Rein.⁸

Results

Tracer tests

In most cases, the model was capable of providing a good fit to the experimental data. In all cases, correlation coefficients

better than 0,99 were obtained. The degree of fit can be assessed by the curves shown in Figure 1, which shows results of the F.C.B. pan tracer tests. The conditions of the experimental tests and the fitted parameters are shown in Table 1.

Where tracer recovery could be calculated, values fairly close to 100% were obtained. Worst agreement was obtained in the Maidstone 'C' pan and Felixton 'A' pan tests, where it is evident that the fitted massecuite volumes also differed significantly from the expected massecuite volumes. Confidence in these two sets of data is somewhat diminished.

Because of differences in average residence time, the comparison shown in Figure 1 is difficult. However, if the axes are

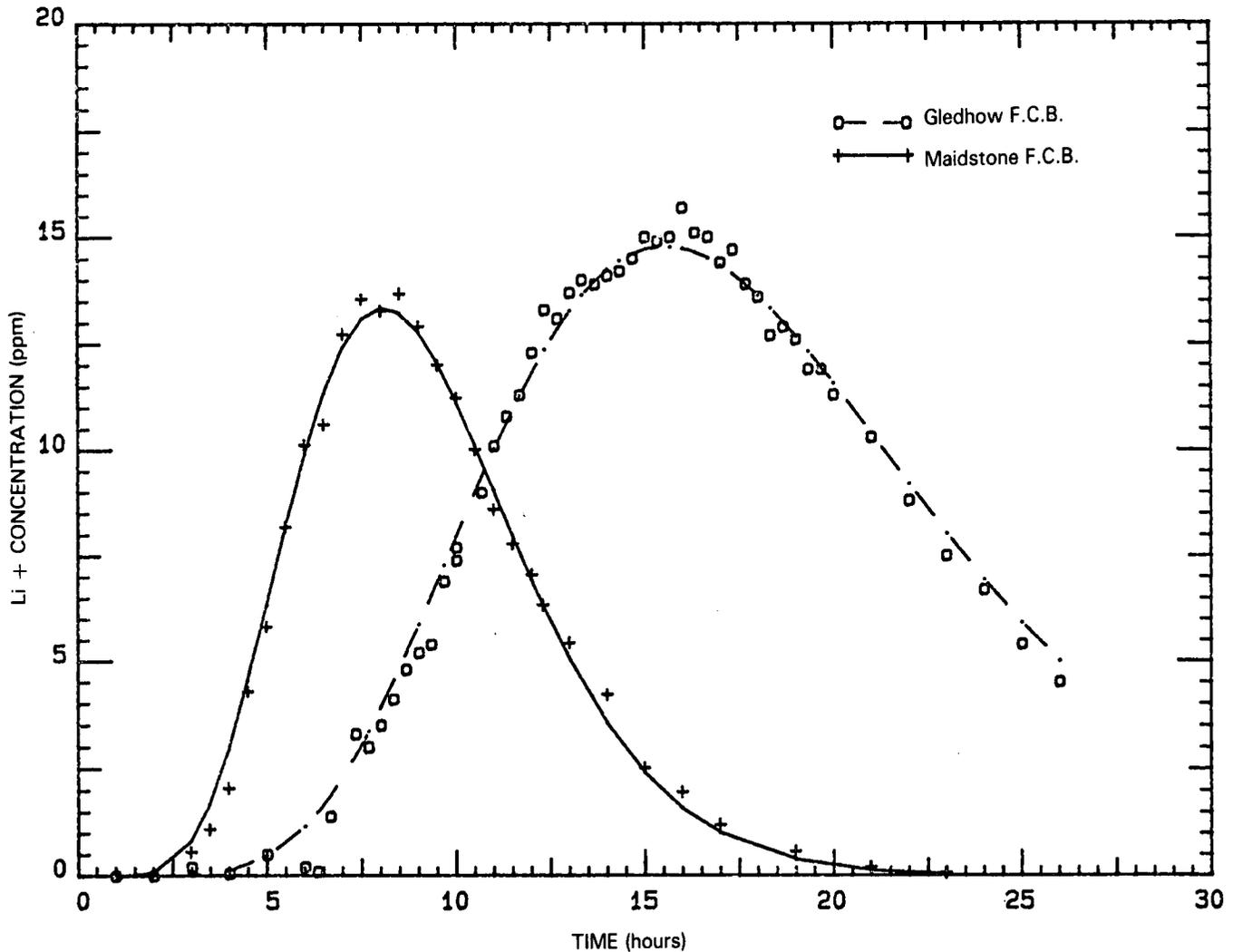


FIGURE 1 Tracer test results reported by the S.M.R.I. on F.C.B. continuous 'C' pans. Lines through the data points represent the best fit of the modified model (equation 2) to the data.

TABLE 1
Results of tracer tests in the South African sugar industry

Mill	MS	GH	GH	ME	AK	MS	MS	MS	FX	FX
Pan type	FCB	FCB	FCB	Mod. Batch	Mod. Batch	THS	THS	THS	THS	THS
Date of test	1980	1980	1981	1980	1981	1983	1982	1982	1984	1985
Massecuite grade	'C'	'C'	'C'	'C'	'A'	'A'	'B'	'C'	'A'	'C'
Pan volume (m ³)	64	104	104	16	50	120	110	110	120	76
No. of compartments	15	14	14	6	6	12	12	12	12	12
Equivalent no. of tanks-in-series	9	9	9	9	5	17	13	23	12	13
Mean residence time (hours)	9,2	17,4	12,5	6,9	1,6	5,1	6,9	11,0	5,2	8,1
Co (ppm. hours)	101,3	232,1	66,4	54,4	12,2	68,6	62,9	132,2	52,65	91,4
Calculated massecuite volume (m ³)	—	—	—	—	44,4	125,0	115,9	75,0	148,9	74,7
Tracer recovery (%)	—	—	—	—	—	98,5	96,2	94,3	108,1	94,1
Correlation coefficient	0,998	0,998	0,990	0,992	0,991	0,999	0,997	0,991	0,996	0,997

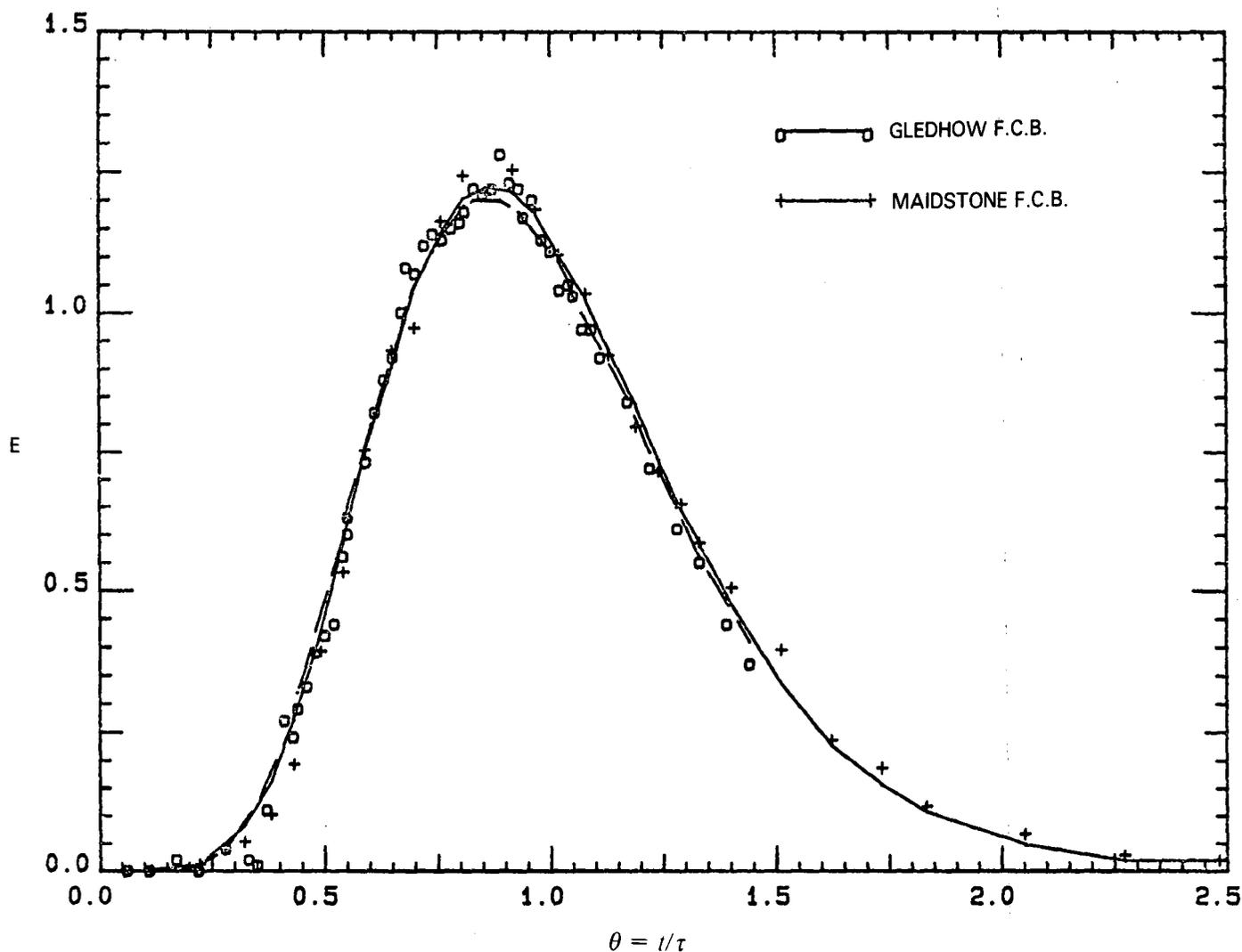


FIGURE 2 Results of tracer tests in Figure 1 shown plotted using dimensionless co-ordinates.

changed to incorporate normalised concentrations and times, the resulting curves become more comparable. The same data in Figure 1 is shown in Figure 2, with the axes defined as follows:—

$$E = C\tau/C_0 \dots \dots \dots (8)$$

$$\theta = t/\tau \dots \dots \dots (9)$$

This shows that the flow characteristics displayed by the F.C.B. pan tests are very similar.

Of interest in Table 1 is the number of tanks-in-series equivalent to the flow system of each pan. The F.C.B. pans are equivalent to 9 well-mixed tanks-in-series, although there are 14 or 15 compartments in the pan. This indicates the presence of significant short-circuiting of massecuite. The long tails on the residence time distribution curves indicate an unusually long hold-up of part of the massecuite in the pan.

Results for the modified batch pans are shown in Figure 3. In this case, the equivalent number of tanks-in-series is similar to, or greater than the number of compartments, which suggest that short-circuiting of massecuite does not occur.

The results from the modified batch pans at Mount Edgecombe and Amatikulu proved that if careful attention is given to the massecuite flow path and particularly the way in which massecuite passes from one compartment to the other, short-circuiting can be prevented.

It is essential that massecuite passing from one compartment to another is prevented, by baffles or some other arrangement, from flowing from the inlet directly to the outlet of the same

compartment bearing in mind the circulation pattern of massecuite within each compartment.

Results of boiling 'A', 'B' and 'C' massecuites, in the Tongaat-Hulett pan at Maidstone, are shown in Figure 4. In this case, the flow system is equivalent to between 13 and 23 tanks-in-series. From this it is concluded that short-circuiting is absent and that, in addition, some measure of plug flow is obtained within each compartment.

There is a significant difference in the equivalent number of tanks-in-series on three different grades of massecuite in the same pan. However, it is likely that the degree of control, the consistency of the massecuite and the evaporation rate in the pan at the time of the test, all contribute to modifying the flow characteristics.

In addition, once the number of tanks is somewhat higher, changes in the number of tanks have a substantially smaller effect on the shape of the residence time distribution curve. Where the number of tanks is low, say 10 or less, a change in n of only 1 unit has a more significant effect.

Results of the tracer tests are shown in Table 2. An interesting fact is that the average residence time is significantly greater than the nominal residence time (where the nominal residence time is expressed as the ratio of pan volume/average massecuite production rate). This is a consequence of the feed of molasses or syrup along the length of the pan and results in the crystal residence time being about 60% higher than the nominal residence time.

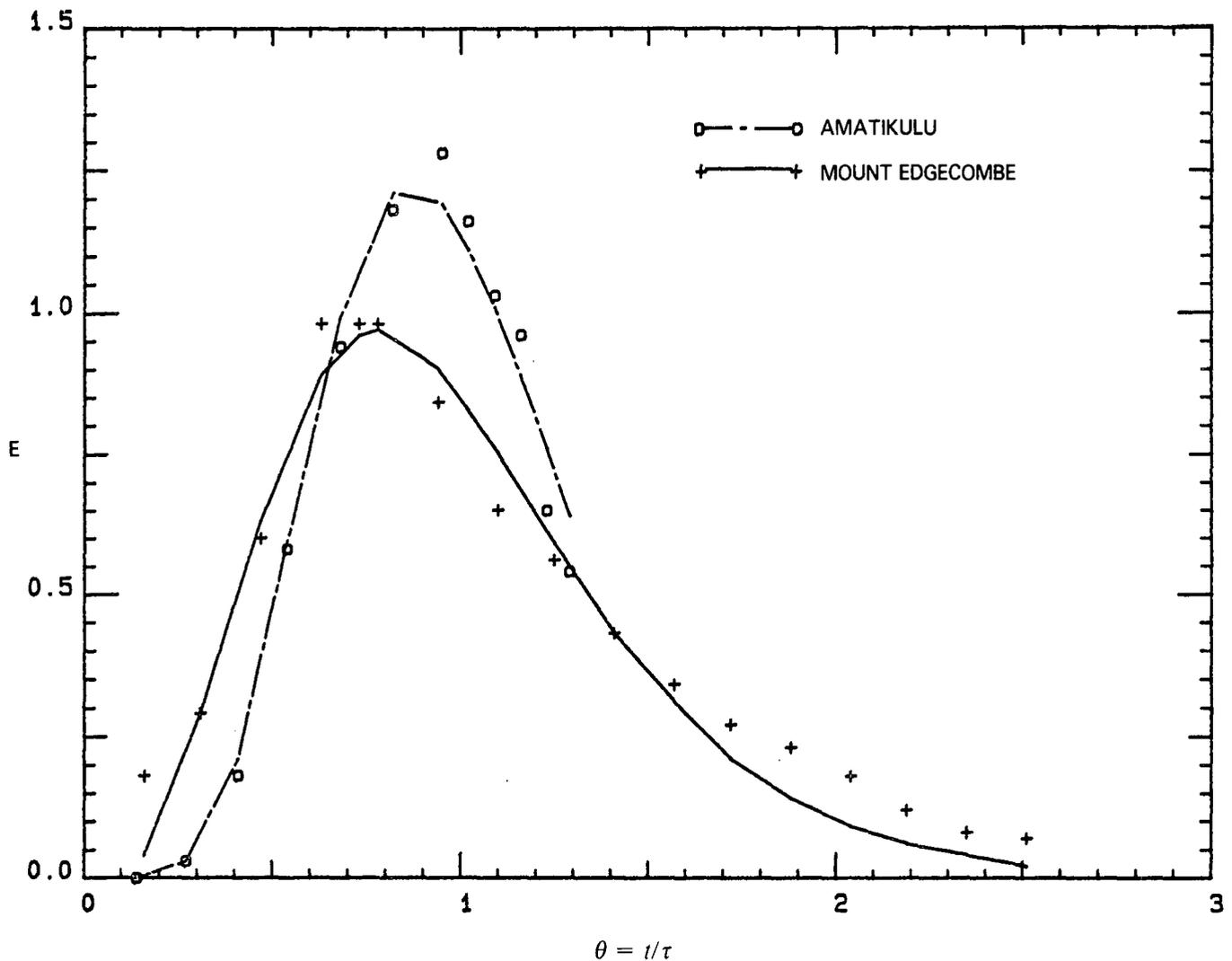


FIGURE 3 Results of residence time distribution measurements in modified batch pans at Amatikulu ('A' massecuite) and Mount Edgecombe ('C' massecuite).

TABLE 2
Massecuite flows & residence times from tracer tests

Mill	MS	GH	GH	ME	AK	MS	MS	MS	FX	FX
Mean residence time (hours)	9,2	17,4	12,5	6,9	1,6	5,1	6,9	11,0	5,2	8,1
Nominal residence time (hours)	5,8	9,9	7,3	4,5	0,91	3,2	4,4	9,3	3,0	4,7
Ratio actual/nominal residence time	1,59	1,76	1,71	1,53	1,76	1,59	1,57	1,18	1,74	1,72
Seed flow rate (m ³ /hour)	3,37	2,31	2,38	1,18	13,5	13,3	9,7	3,2	18,7	4,1
Massecuite flow rate (m ³ /hour)	11,0	10,5	14,3	3,58	40,9	38,0	25,3	11,8	39,0	15,9
Massecuite/seed ratio	3,26	4,55	6,00	3,03	3,03	2,86	2,61	3,69	2,09	3,88

It is assumed that the tracer in the mother liquor has the same residence time distribution as the crystals. The same reasons which preclude the use of a classified product removal, that is high viscosity of mother liquor and a small difference between the densities of crystal and mother liquor, probably ensure that this assumption is correct and that crystal and mother liquor residence times are identical.

In some of the tests, samples were taken at intermediate points along the pan to see whether any difference in flow characteristics in the early or later part of the pan could be distin-

guished. An example of the results obtained is shown in Figure 5. The relationship between number of compartments and equivalent number of tanks-in-series indicates that the flow behaviour is fairly uniform throughout the pan.

A comparison was made between the number of tanks-in-series obtained from the original tanks-in-series model, Equation 1 and the modified model used here, Equation 2. In spite of the physical system implied in the simple model of Equation 1, the use of the simplified model in general, gave fairly similar results, with the best value of the equivalent number of tanks-in-series generally not varying by more than 1 or 2 units.

Changes in the crystal size distribution

The effect of continuous pan operation on crystal size distribution was established by analysing samples of massecuite produced. A number of samples of seed massecuite and product massecuite have been taken from both continuous 'A' pans and batch 'A' pans, to compare the effect of the different mode of operation. Most of the work has been done on the continuous 'A' pan at Maidstone.

During the test periods, the average crystal size produced was low, so that the fines content (expressed as sugar less than 600 microns) was generally high. A better method of assessing size distribution is to look at the coefficient of variation (CV = 100 × standard deviation/mean size). Results obtained in 'A' massecuites on batch and continuous pans are shown in Table 3.

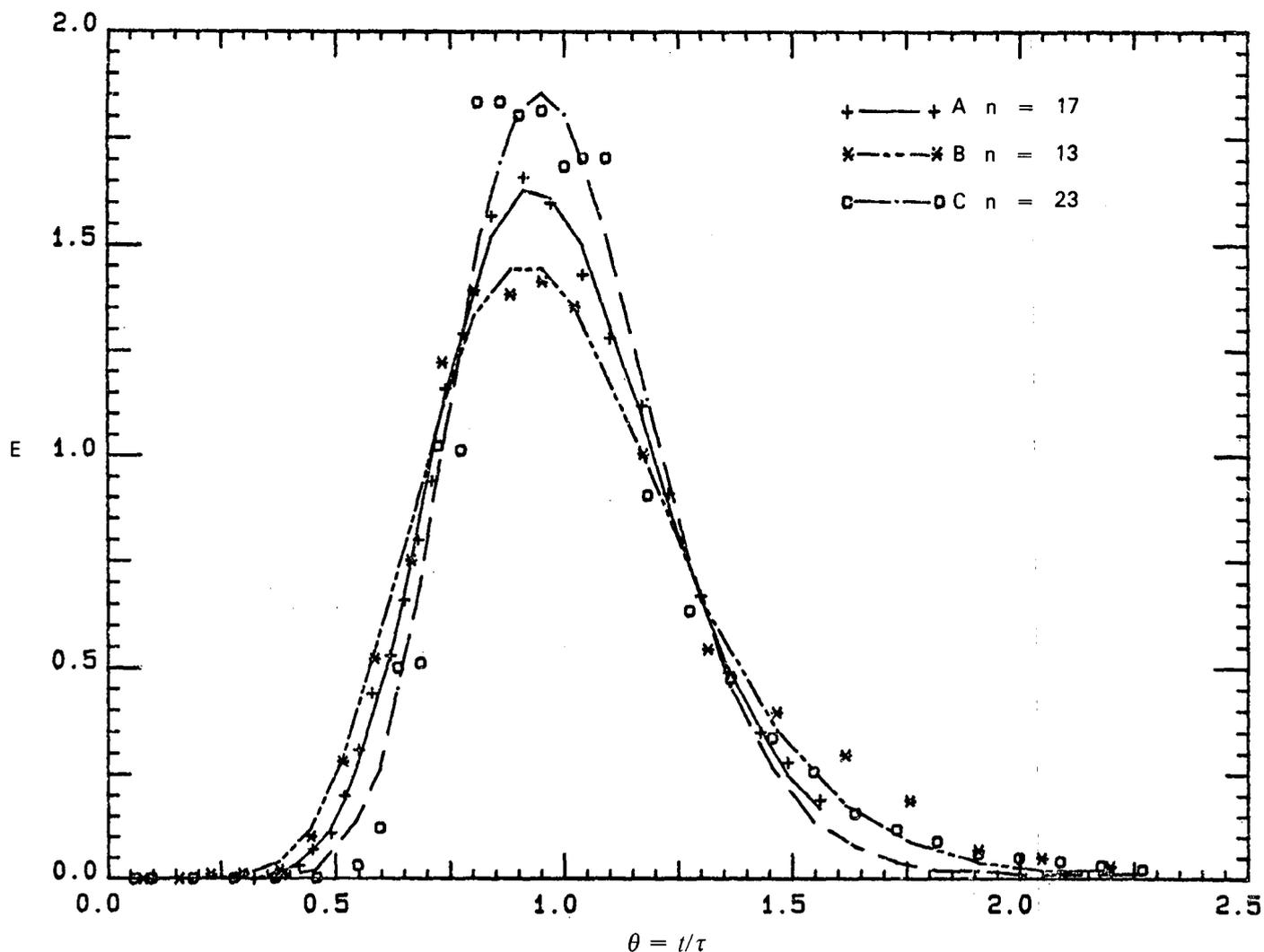


FIGURE 4 Residence time distribution measured in the Maidstone Tongaat-Hulett continuous pan boiling 'A', 'B' and 'C' massecuites.

TABLE 3

Values of crystal mean aperture & coefficient of variation measured in batch & continuous 'A' massecuite pans

Mill	Seed		Massecuite		Change in CV	No. of sets of data
	MA (mm)	CV	MA (mm)	CV		
DL (batch pan)	0,50	37,6	0,61	28,8	8,8	6
DL (batch pan)	0,58	32,9	0,67	27,9	5,0	6
AK (seed only)	0,48	40,6	—	—	—	6
AK (batch pans)	—	—	0,63	33,3	7,3	8
AK (continuous pan)	—	—	0,60	33,7	6,9	9
ME (batch pans)	—	—	0,70	27,9	—	5
MS (THS continuous pan) 1982/83	0,43	39,9	0,61	33,4	6,5	13
MS (THS continuous pan) 1983/84	0,43	33,1	0,61	26,5	6,6	7
FX (THS continuous pan)	0,39	39,0	0,61	36,0	3,0	2

Contrary to reports elsewhere that continuous pans always lead to a worsening of crystal CV, these results show clearly that an improvement in CV from seed to massecuite was always achieved in the Tongaat-Hulett continuous pans. In addition, the improvement was of the same order as that achieved in the batch pans.

Austmeyer¹ has shown that discrepancies between theory and practice are reduced in the case of continuous pans, indicating more uniform conditions in the continuous pan where circulation is better and massecuite heads are lower. Thus, if perfect plug flow could be achieved in a continuous pan, the crystal size distribution could be better than that achieved in a batch pan.

It was shown at Maidstone that affinated crystal quality from the continuous 'A' pan is superior to that from batch boiling.⁴ This supports the view that conditions for crystal growth are better in continuous than in batch pans. Although the flow characteristics are not those of perfect plug flow, the improved crystallisation conditions possibly make up for the spread in crystal residence times which is achieved in practice.

It is interesting to compare how theoretical predictions outlined earlier, correspond with these measurements. Considering only the time dispersion effect, the predicted massecuite CV values as a function of seed CV are shown in Figure 6, calculated from equation (4). These are calculated for various numbers of tanks-in-series, with a value of w of 0,04 mm/hour and a crystal residence time of 5 hours, representing average conditions for the Maidstone 'A' pan. These results show that for an average seed CV of about 35 and $n = 16$, a product CV of about 23 is predicted, ie an improvement of 12 units. The number of tanks-in-series would have to be reduced to 4 to achieve the observed improvement of only 6 units in CV.

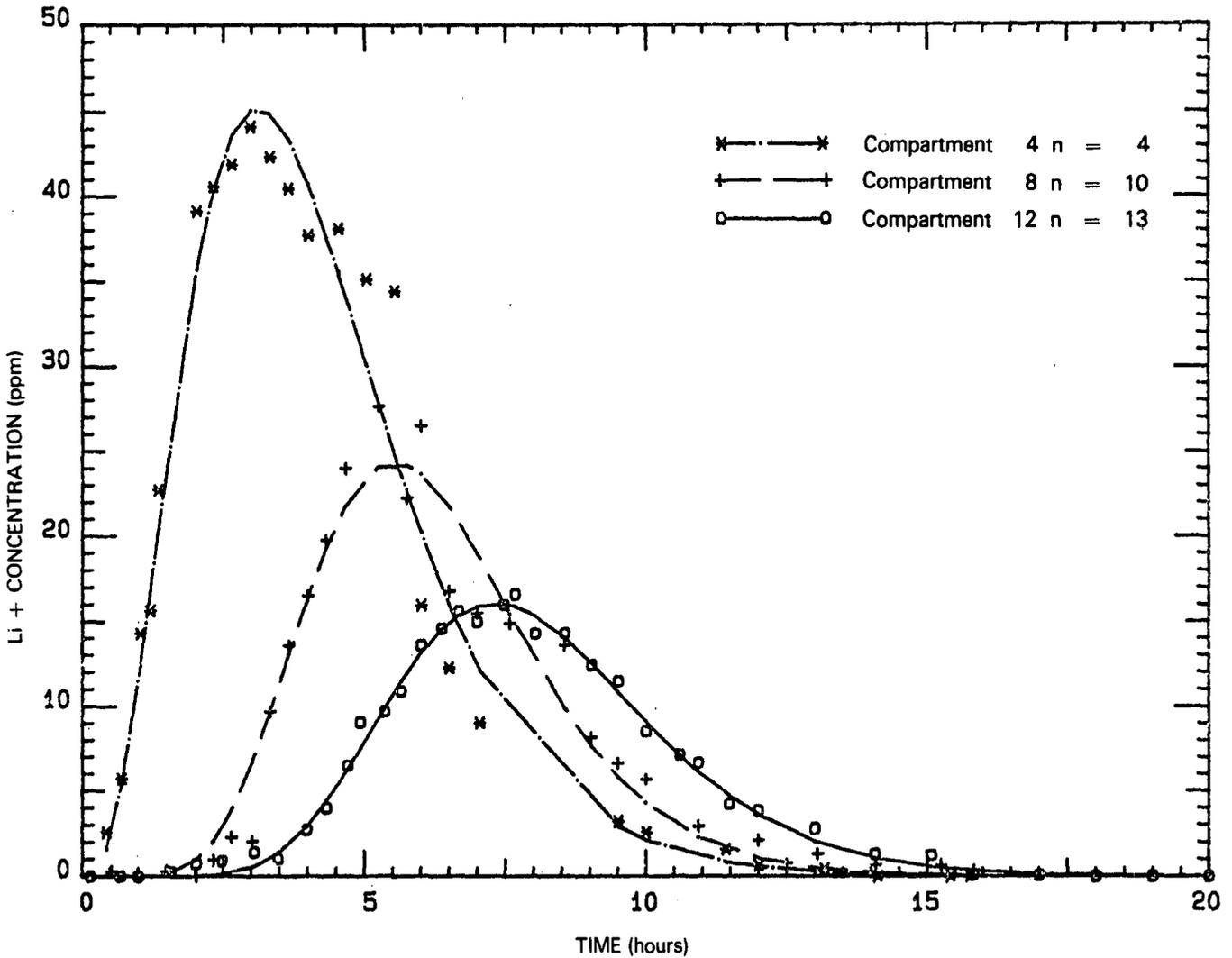


FIGURE 5 Residence time distribution after 4, 8 and 12 compartments in the Felixton 'C' pan.

Clearly, the residence time distribution effect is not sufficient to explain the observed results. Additional dispersion effects of the type suggested by Wright & White¹¹ also need to be included. To include these dispersion effects, the parameter p in equation (7) needed to be evaluated. This was done by comparing the predictions of equation (7) with data obtained from the Maidstone 'A' pan, shown in Figure 7.

The experimental data shown are the averages of 9 sets of experimental data. Assuming the pan to be equivalent to 18 tanks-in-series (ie 1,5 tanks equivalent to each compartment), the equations (3) and (7) were found to fit the data quite well with $w = 0,039$ mm/hour and $p = 0,044$ mm.

Using these values in equation (7), the predictions can be compared with actual data measured at Maidstone, as shown in Figure 8. This suggests a bigger improvement in CV from seed to massecuite when the CV of the seed is high and very little improvement at low CV values. The slope of the line is less than that of the linear regression equation below and this is also shown in Figure 8:—

$$CV(\text{massecuite}) = 0,70 CV(\text{seed}) + 4,565 \dots \dots (10)$$

In this model, a constant growth rate is assumed and this may not be entirely valid since mother liquor purity and brix change throughout a pan, and that growth rate is independent of size. Thus, it is suggested that this model may be useful in looking at the relative magnitudes of size dispersion effects, but is unlikely to yield accurate results.

The introduction of the additional dispersion effects reduces the relative effect of the reduced number of tanks-in-series. From equation (7) it can be seen that the ratio of size dispersion resulting from residence time distribution to other growth dispersion effects is given by $(w^2 \tau^2/n)/(pw\tau) = w\tau/pn$. Using values of p and w evaluated from the Maidstone results, it can be calculated that this ratio is 1,13 for 4 tanks-in-series indicating that the variance due to time distribution effects is just slightly more than that due to other effects. However, when $n = 16$, the dispersion due to time distribution is only just over a quarter of that due to the other effects.

All these results apply only to the kind of conditions experienced in South African 'A' massecuites. Of interest is the value of p obtained. Based on results given by Wright & White,¹¹ a value of p roughly double the value of 0,044 evaluated here for this purity and growth rate can be expected. This could again be indicative of the good homogeneous crystallisation conditions achieved in continuous pans.

The significance of these results is that attention given to good circulation and control in the pans, which can affect the magnitude of the "other" size dispersion effects, is at least as important as the attention given to achieving good plug flow characteristics. In this respect, the smaller improvement in the CV in the two sets of data shown for the Felixton 'A' pan in Table 3 could be partly due to the poor control in operation at the time the measurements were taken.

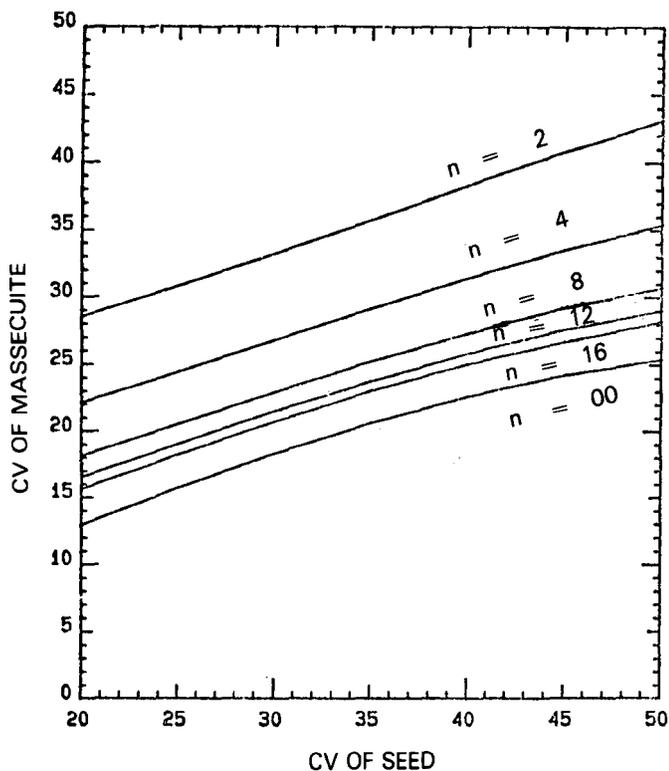


FIGURE 6 Predicted massecuite crystal CV as a function of seed crystal CV and number of tanks, neglecting other growth dispersion effects, from equation (4).

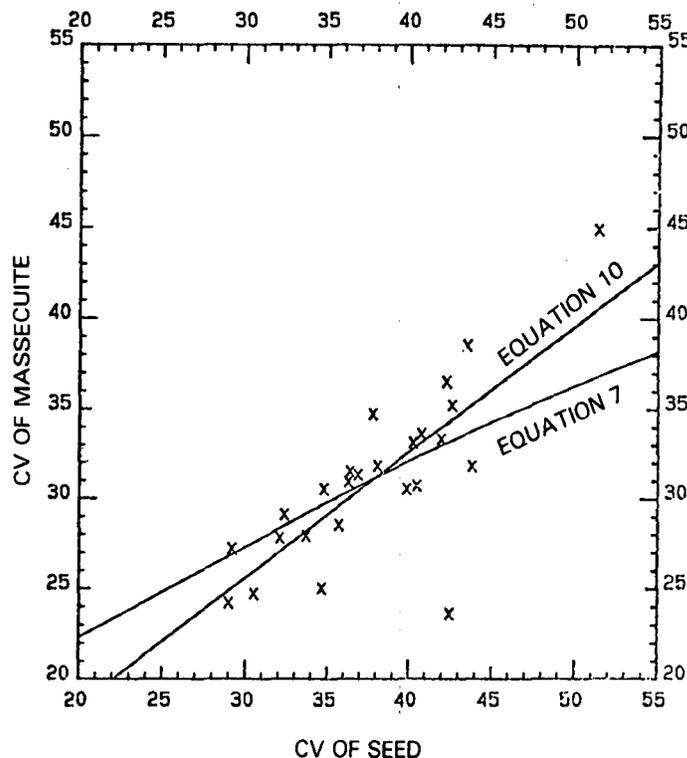


FIGURE 8 Measured values of crystal CV in seed and massecuite from the Maidstone continuous 'A' pan, compared with predictions of the model (equation 7) and the least squares linear regression (equation 10).

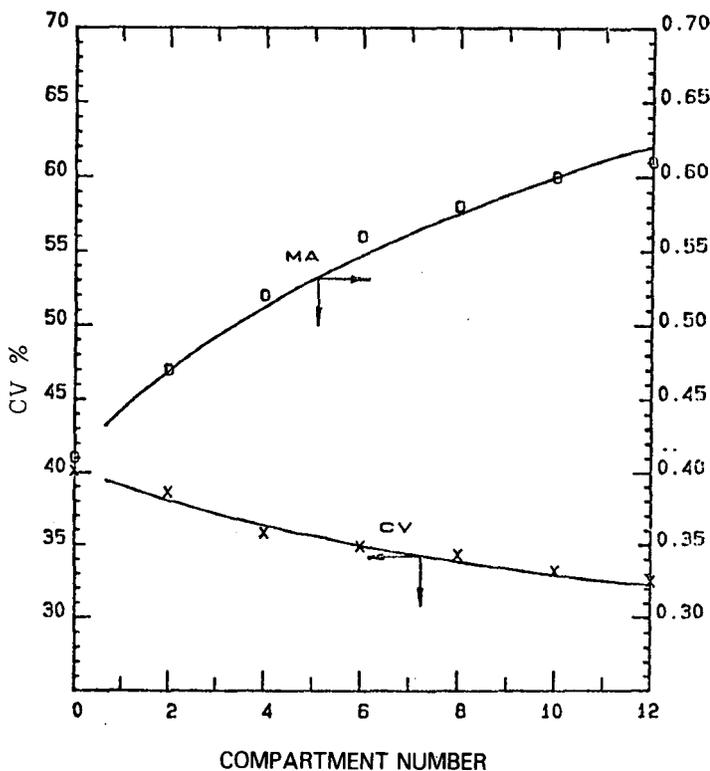


FIGURE 7 Comparison of measured values of mean aperture and CV from the Maidstone continuous 'A' pan with equations (3) and (7) to evaluate average growth rate w and dispersion parameter p . ($w = 0,039$ mm/hour and $p = 0,044$ mm).

Very little other data with which to compare these results, is available. Results given by Austmeyer¹ on an F.C.B. 7-compartment continuous pan, yielded a sugar with a CV of 45%, compared with a CV of 27% obtained on batch boiling. Based on the tracer tests on F.C.B. pans in South Africa, the 7-compartment pan could correspond to something less than 7 well-mixed tanks-in-series. Under these conditions, the crystal size distribution is severely affected.

A new BMA continuous pan which produced white sugar was installed at Wabern in Germany and has been in operation for two beet campaigns. It consists of 4 well-stirred pan compartments, one above the other. Crystal quality is good because the boiling conditions are well-controlled. However, in this system, which probably approximates 4 well-mixed tanks-in-series, the sugar produced has a CV of around 40%, whereas sugar produced by batch pans has a CV of about 20% to 25%.¹⁰

Very few results have been reported on the effect on crystal size distribution in 'C' massecuite pans. This is surprising in view of the length of time that low grade continuous pans have been operating. For lower purity massecuites, values of w are generally lower. However, the results of Wright & White¹¹ indicate that p will also be lower. Thus, in order to assess the relative importance of time distribution to other size dispersion effects, some additional measurements on 'C' massecuites will be necessary.

It would appear from all these results that it is desirable to have a continuous pan flow system equivalent to at least 12 tanks-in-series, in order to achieve a satisfactory improvement in crystal CV during the boiling process. If the flow system is closer to plug flow, it is possible that a crystal size distribution better than that achieved in batch pans can be obtained.

Conclusions

A mathematical model has been developed to represent the flow system in continuous vacuum pans to account for the feed of syrup or molasses along the length of the pan. This model is useful in quantifying the flow characteristics of the continuous pan system. Pans installed recently in Tongaat-Hulett mills have flow characteristics which result in crystal size distribution 'A' massecuites which is at least as good as that produced in batch pans.

Attempts to predict theoretically how crystal size distribution is affected by residence time distribution have been partially successful. However, it appears that size distribution is also affected to a large extent by conditions in the pan. The attain-

ment of more uniform conditions in a continuous pan is an advantage over batch pans in this respect.

It appears, therefore, that in any application of continuous pans where product crystal size distribution is important, it is desirable to use a seed with a good CV, to have a flow system equivalent to about 12 (or more) tanks-in-series and to have good circulation and uniform boiling conditions in the pan. Under these conditions, size distributions equal to or better than those achieved in batch pans, should be possible.

Acknowledgements

A large number of people have participated in the investigation into continuous pans. In particular, the assistance of the staff at Mount Edgecombe, Amatikulu, Maidstone and the new Felixton mill is greatly appreciated. The Operations Research Department assisted with much of the computer work and Mr R. G. Hoekstra helped evaluate the size distribution data. The development of the mathematical model was largely due to the efforts of a vacation student, Steven Brown.

Nomenclature

C	Concentration of tracer	(ppm)
C_0	Concentration scaling factor	(ppm hours)
CV	Coefficient of variation = $100 \sigma/\mu$	
E	Residence time distribution function (Equation 8)	
MA	Mean aperture	(mm)
n	Number of tanks in tanks-in-series model	
p	Dispersion parameter defined in equation (5)	(mm)
s	Parameter used in Laplace Transform operations	
t	Time	(hours)
V_T	Masseccuite volume in pan	(m^3)
V	Volume of one theoretical tank	(m^3)
v_0	Seed masseccuite flow into pan	(m^3 /hour)
v_n	Masseccuite flow out of pan	(m^3 /hour)
w	Effective crystal growth rate	(mm/hour)
x	Feed to each tank	(m^3 /hour)
μ_m	Crystal mean aperture size	(mm)
$\mu_r \cdot \mu_i$	Means size (number basis) of crystals in masseccuite entering and leaving pan	(mm)
$\sigma_r \cdot \sigma_i$	Standard deviations of crystals in masseccuite entering and leaving pan	(mm)

θ	Dimensionless time = t/τ	
τ	Mean residence time	(hours)
λ_i	Mean frequency of passing for each fluid element at the i th tank	(hours ⁻¹)

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APPENDIX I

DERIVATION OF MODIFIED TANKS-IN-SERIES MODEL - STOCHASTIC APPROACH

Stochastic models attempt to account directly for the random nature of the system. The *Markov Property* allows important simplification in describing the time behaviour of a random (stochastic) process.

The main point about the *Markov Property* is:—

Only the last state occupied by the process is relevant in determining its future behaviour, that is, the probability of making a transition to any state depends only on the state presently occupied.

Assumptions

The flow system is represented by a model in which:—

- equal amounts (x) of liquid are fed into each theoretical tank.
- the sum of the volumes of each theoretical tank equals the total volume of the pan, that is there is no "dead space".
- mixing is complete throughout each theoretical tank.

From assumption (i) above, each tank has a volumetric flow through it, according to an arithmetic progression (where x is the common difference).

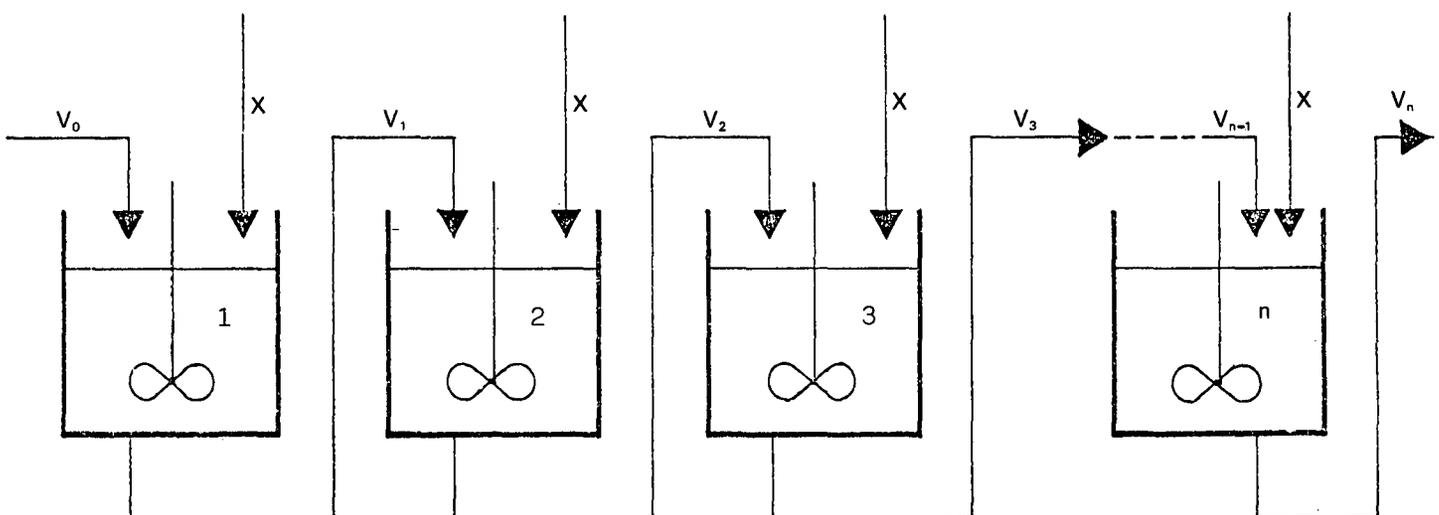


FIGURE 9 Schematic representation of flow model

$$\text{Total volume } V_T = \sum_{i=1}^n V_i \quad (V_1 = V_2 = V_3 = \dots = V)$$

$$\therefore VT = nV \quad \dots \dots \dots (11)$$

The total volumetric flow out of the system is the volumetric flow out of the nth tank.

$$v_n = v_o + nx \quad \dots \dots \dots (12)$$

The volumetric flow out of the ith tank is:—

$$v_i = v_{i-1} + x$$

$$\therefore v_i = v_o + ix \quad \dots \dots \dots (13)$$

Combining equations (11) and (12) leads to:—

$$v_i = v_n - (n - i)x$$

$$i = 0, 1, 2 \dots n \quad \dots \dots \dots (14)$$

The mean residence time at the ith tank is:—

$$\tau_i = V/v_i$$

Then the mean frequency of passing for each fluid element at the ith tank is:—

$$\lambda_i = 1/\tau_i = v_i/V$$

$$i = 1, 2 \dots n \quad \dots \dots \dots (15)$$

Consider the following approach to the variation in tracer concentration. A tracer element enters the first tank at time $t = 0$ and stays there for a random time. It then moves to the next tank and stays there for an additional random time before it moves to the following tank. Thus, for a given time t , the tank which the element may occupy is a random variable. Also, for a fixed tank, the time for the tracer element in that tank is a random variable.

Let $P_i(t)$ be the probability that a tracer element is found in the ith tank at time t . Then the probability that an element is in the ith tank at $(t + dt)$ is equal to the sum of two probabilities. That is the probability at time t in the $(i - 1)$ tank multiplied by the probability of transferring from the $(i - 1)$ tank to the ith tank between time t and $(t + dt)$, and the probability at time t in the ith tank multiplied by the probability that the element remains in the same ith tank between t and $(t + dt)$.

$$\therefore P_i(t + dt) = (\lambda_{i-1} dt) P_{i-1}(t) + (1 - \lambda_i dt) P_i(t) \quad \dots \dots \dots (16)$$

Rearranging and dividing by dt :—

$$\therefore \frac{dP_i(t)}{dt} = -\lambda_i P_i(t) + \lambda_{i-1} P_{i-1}(t) \quad \dots \dots \dots (17)$$

Noting the initial conditions:—

$$P_i(0) = \begin{cases} 1 & i = 1 \\ 0 & i = 2, 3 \dots n \end{cases}$$

The Laplace Transform technique is used to evaluate equation (17):—

$$P_i(s) = \frac{1}{s + \lambda_i} \quad \dots \dots \dots (18)$$

$$\frac{P_i(s)}{P_{i-1}(s)} = \frac{\lambda_{i-1}}{s + \lambda_i} \quad i = 2, 3 \dots n \quad \dots \dots \dots (19)$$

$$\text{Also, } P_i(s) = \frac{P_i(s)}{P_{i-1}(s)} \cdot \frac{P_{i-1}(s)}{P_{i-2}(s)} \cdot \frac{P_{i-2}(s)}{P_{i-3}(s)} \dots \frac{P_2(s)}{P_1(s)} \cdot P_1(s) \quad \dots \dots (20)$$

Using equations (18) and (19) in (20) yields:—

$$P_i(s) = \frac{\lambda_{i-1} \lambda_{i-2} \dots \lambda_1}{(s + \lambda_i)(s + \lambda_{i-1}) \dots (s + \lambda_1)} \quad \dots \dots \dots (21)$$

$$i = 1, 2 \dots n$$

The Inverse Laplace transform of equation (21) is obtained with the use of Cauchy's Residue Theorem and the result is:—

$$P_i(t) = \prod_{m=1}^{i-1} \lambda_m \left(\sum_{j=1}^i \frac{\exp(-\lambda_j t)}{\prod_{\substack{k=1 \\ k \neq j}}^i (\lambda_k - \lambda_j)} \right) \quad \dots \dots \dots (22)$$

An expression for $E(t)$ can be obtained by noting that the probability of the random exit time T in the time interval $(t, t + dt)$ is equal to the probability of the tracer element being in the last tank n at time t , multiplied by the probability of it transferring from the last tank to the outlet during the interval $(t, t + dt)$.

$$P(t < T < t + dt) = P_n(t) \cdot \lambda_n dt \quad \dots \dots \dots (23)$$

But $E(t) dt = P(t < T < t + dt)$

$$\therefore E(t) dt = P_n(t) \lambda_n dt$$

Using equation (22):—

$$E(t) = \prod_{j=1}^n \lambda_j \sum_{j=1}^n \left(\frac{\exp(-\lambda_j t)}{\prod_{\substack{k=1 \\ k \neq j}}^n (\lambda_k - \lambda_j)} \right) \quad \dots \dots \dots (24)$$

From equations (14) and (15) we can write:—

$$\lambda_i = (v_n - (n - i)x)/V \quad \dots \dots \dots (25)$$

Inserting equation (25) into (24) yields:—

$$E(t) = \prod_{i=1}^n (v_n - (n - i)x)/V \cdot \sum_{j=1}^n \frac{\exp(-(v_n - (n - j)x)t/V)}{\prod_{\substack{k=1 \\ k \neq j}}^n \left(\frac{v_n - (n - k)x}{V} - \frac{v_n - (n - j)x}{V} \right)}$$

This can be modified to:—

$$E(t) = \prod_{i=1}^n (v_n - (n - i)x)/V \cdot \sum_{j=1}^n \frac{\exp(-(v_n - (n - j)x)t/V)}{\prod_{\substack{k=1 \\ k \neq j}}^n ((k - j)x/V)} \quad \dots \dots (26)$$

Therefore, the system can be described by equations (11), (12) and (26).

APPENDIX II

RELATIONSHIPS BETWEEN MEANS & VARIANCES ON A NUMBER COUNT & MASS BASIS ASSUMING A LOG-NORMAL DISTRIBUTION

In order to convert mass distribution parameters (subscript m) to a number distribution basis:—

$$\mu = \mu_m (\sigma^2/\mu^2 + 1)^{-3}$$

$$\sigma^2 = \sigma_m^2 (\sigma^2/\mu^2 + 1)^{-6}$$

The inverse relationships are:—

$$\mu_m = \mu (\sigma^2/\mu^2 + 1)^3$$

$$\sigma_m^2 = \sigma^2 (\sigma^2/\mu^2 + 1)^6$$