OPTIMISATION OF THE CRUSHING PROGRAMME FOR THE SEASON BY DYNAMIC PROGRAMMING

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Abstract

The application of the technique of dynamic programming by computer to the optimisation of the season's crushing programme of a mill is described. Within any given contraints on factory throughput rates, cane availability and starting and finishing dates for the season, the computer program will specify the starting date, month-by-month crushing rates and the finishing date. Expected monthly values of process variables and time utilisation, the drop in overall recovery with increased throughput rate and the various types of costs are taken into account. The output includes the monthly performances, efficiencies and costs, as well as the final, maximised profit for the season. An example of its use is given.

1. Introduction

In planning the crushing programme for the year, mill management have to decide when to start, at what rate to crush each month, and when to finish. Their decision must take into account the following:-

* Constraints on the factory throughput rates
* Constraints on the supply of cane to the mill
* Changes in the analyses of the cane and their effects on maximum throughoutput rates, overall recoveries and sugar production
* The drop in recoveries with increase in throughput rate
* Expected time efficiencies

To assist management in making such decisions, the Crushing Rate Optimisation Program (CROP) was developed for running on a computer.

2. Use of Dynamic Programming

The unknowns to be determined in this problem are the crushing rates for each individual month, and the number of weeks crushed in each month. Each month is considered to consist of 4,33 weeks, and obviously all months occurring in the middle of the crushing season will have 4,33 weeks of crushing, but the starting and finishing months of the season would, in general, consist of fewer crushing weeks than 4,33.

The optimisation technique used for determining the values of these variables is known as dynamic programming. This is a search technique which, instead of doing an exhaustive search of all the millions of possible combinations of values of the variables, eliminates most of the possibilities, so that eventually only a small fraction of the total number of combinations need be calculated. This technique is completely different from linear programming, and its advantages are that non-linear (i.e. non-straight line) relationships pose no problems whatsoever, and that the written computer program is completely self-contained, i.e. there is no need to interface it with a special linear programming package.

The dynamic programming technique lends itself to optimisation of systems with stagewise processes, where the state of the system at the end of each stage can be fully described by only a small number of different variables – the fewer the better. By "stages" one could either mean successive technical processes which the material has to undergo or, as in this application, successive periods of time over which the material has to be processed. In this application each stage is considered to be one calendar month. Fig. 1 illustrates the situation for Month i, where $i = 1, 2, \ldots, n$, for a programme over n months.

At the commencement of any Month i, a decision has to be made on the rate at which the cane has to be crushed. This is called the *decision variable* D_i, and is defined as the tons of cane crushed per week. For ease of programming, this decision variable was not considered as a continuous variable, but was expressed in multiples of 1 000 tons cane per week. For planning purposes, and in view of the margin of error in many of the assumptions, such accuracy is more than adequate.

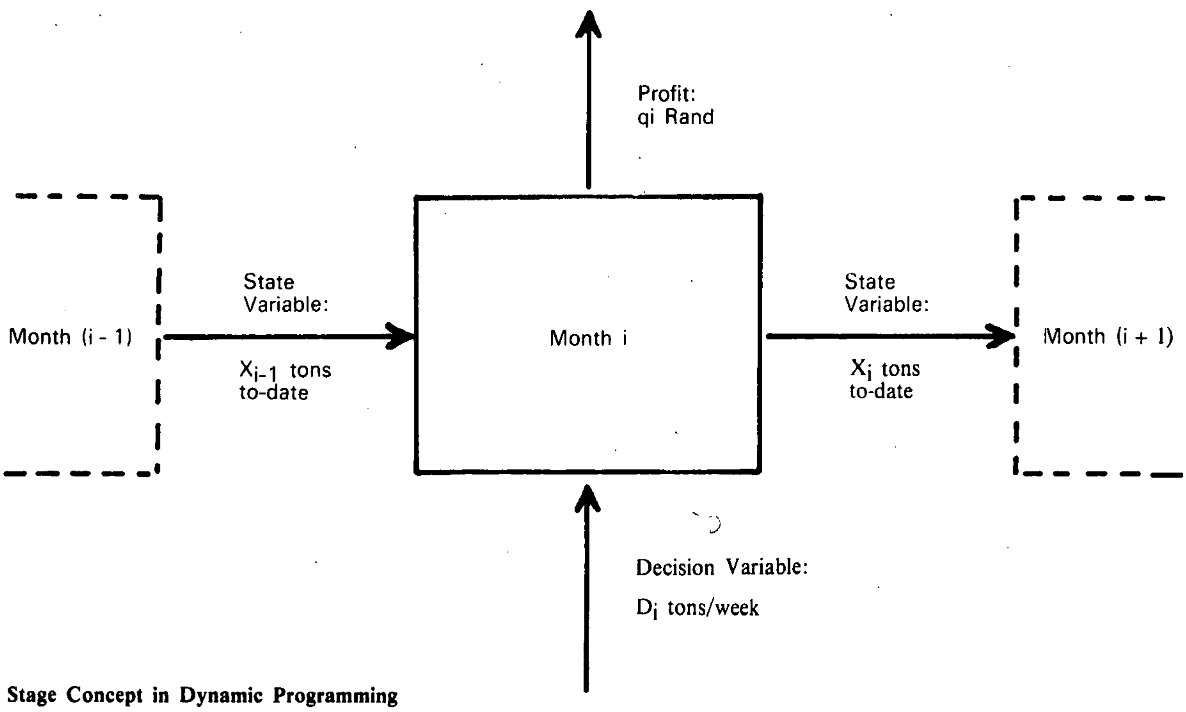

Figure 1: The Stage Concept in Dynamic Programming

At the end of any Month i, the *state variable* X_i should express the state of the system, in so far as it affects subsequent decisions D_{i+1}, D_{i+2}, ... which have to be made. In this application, the task is to crush a certain amount of cane over the season, and at the end of each Month i to consider in what monthly pattern the remaining cane has to be crushed over the remainder of the season. All that need be known therefore is the amount of cane X_i which had already been crushed to date; how that cane had actually been crushed is irrelevant to the decision of how to crush for the remainder of the season. The state variable is therefore defined as the total amount of cane (expressed in multiples of 4 333 tons) which had been crushed to date.

Actually there is another state variable, which specifies whether the month under consideration is a part crushing month (such as would be for a starting or finishing month) or a full month (like one would find in the middle of the season), but for the purposes of explanation this has been ignored.

Another characteristic of dynamic programming is that the profit q_i arising from any given Stage i should be dependent on the decision variable D_i of only that and not any other stage. Here this is indeed the case, because the profit q_i arising in Month i will depend only on the amount of cane crushed D_i in that month.

A further requirement is that the profits $q_1, q_2, \ldots, q_n$, of the individual stages should be additive to give the profit of the entire system. This also applies to the monthly profits in our example, which add up to the total profit for the season, which is to be maximised.

In general, after any Stage i, a number of combinations $(D_1, D_2, \ldots, D_i)$ of the decision variable over the Stages 1, 2, ..., i, will all result in the same value of variable X_i. The principle of dynamic programming is to retain, for each value of state variable X_i, only that combination $(D_1, D_2, \ldots, D_i)$ of the decision variables which provided the maximum value of cumulative profit $q_1 + q_2 + \ldots + q_i$, discarding all other $(D_1, D_2, \ldots, D_i)$ combinations from further consideration and thus reducing the length of search.

As an example of the power of this system of elimination, consider a process of 9 stages and 10 possible values of decision variable per stage. Assuming no constraint on the state variable X_9 at the end of the process, an exhaustive search would involve a "tree" ending in 10^9 branches, i.e. there would be 1 000 000 000 combinations of $(D_1, D_2, \ldots, D_9)$ to calculate.

Application of dynamic programming to the same problem would entail 10, 20, ..., 90 different values of state variables $X_1, X_2, \ldots, X_9$ after stages 1, 2, ..., 9 respectively. Each state variable X_i would involve a search over 10 values of decision variable D_i, giving a total number of combinations of 10(10 + 20 + ... + 90)

$$= 10(10 + 90) \times 10/2$$
$$= 5000$$

The number of searches is therefore reduced by a factor of $10^9 \div 5000 = 200\,000$.

For further details on the technique of dynamic programming, text books such as that by Nemhauser[1] can be consulted.

3. Recovery Considerations

3.1. *As a function of laboratory analyses:*

In the sugar industry, we have a general relationship – Overall Recovery = Extraction x Boiling House Recovery/100.

Each of the above 2 factors can be expressed in terms of the laboratory analyses and calculated performance yardsticks.

According to Smith[2]

$$\text{Extraction} = 100 - \frac{(100 - C)\, F_{bc}}{0.03936\,(100 - F_c)\, P_c 0.6} \quad \ldots\ldots\ldots (1)$$

Where C = Corrected reduced extraction (CRE)
F_c = Fibre % cane
F_{bc} = Fibre % bagasse in cane
P_c = Pol % cane

In Appendix 1, it is shown that Boiling House Recovery

$$BHR = 100 - L_c - L_u - 100HR \bullet \left(\frac{T + 39.94 - 19.6 \log G}{100 - (T + 39.94 - 19.6 \log G)}\right)\left(\frac{100 - J}{J}\right) \quad \ldots (2)$$

where L_c = Filter cake loss, pol in filter cake % pol in mixed juice
L_u = Undetermined loss, pol lost % pol in mixed juice
H = Pol/Sucrose ratio in molasses
R = Impurity recovery ratio, non-sucrose in molasses/non-pol in MJ
T = Target purity difference (TPD) by SMRI revised formula
G = Reducing sugar/ash ratio in molasses
J = Purity of mixed juice, %

It is therefore possible to calculate, month-by-month, the extraction and the boiling house recovery as a function of the cane analysis and the operating conditions.

Exhibit 1 shows typical input data used in the program, the first part of which concerns the process data. It is important that average or expected values, smoothed out from month to month, are provided. It would be senseless to use historical data of only one year, because it is highly unlikely that the deviations of that year would be repeated.

3.2. *Effect of throughput rate on extraction:*

In the expression for extraction, CRE is, by definition, a measure of extraction plant performance, whereby the cane quality effects have been eliminated. It is assumed that the CRE will be affected by throughput rate, in tons fibre per hour, as follows:

Up to a certain fibre rate, called the reference tons fibre/hour, the CRE will remain constant at a value called the reference CRE, and for fibre rates beyond the reference tfh, the CRE will drop by a given amount for every additional ton fibre per hour. This relationship is illustrated in Fig. 2, and is used for the CRE term in expression (1) for extraction.

3.3. *Effect of throughput rate on boiling house recovery:*

In expression (2) for BHR, it is the performance yardsticks TPD, filter cake loss and undetermined loss which represent the way the plant is operated. If undetermined loss and filter cake loss are not affected by cane throughput rate, TPD has to be expressed as a function of throughput rate, which might be tons/hour non-sucrose in mixed juice or tons brix/hour in mixed juice, or both. It is assumed that, starting with low rates, the TPD remains at a constant value called the reference TPD, until the reference tons non-sucrose per hour and/or the reference tons brix per hour had been reached, beyond which the TPD rises at a given rate for every additional ton brix per hour and/or ton non-sucrose per hour.

The above relationships were substituted into the expressions for extraction and boiling house recovery, so that the overall recovery is a function of throughput rates as well as cane quality and other operating conditions.

4. Constraints

4.1. *Factory Constraints:*

For each month, the program obeys the following upper constraints on throughput rates:

OPTIMIZATION OF MILL CRUSHING PROGRAM **DATE: 13/04/78** **TIME: 10/44/33**

MILL: EXAMPLE MILL **SEASON: 1978/79** **RUN No: 1**

OBJECT: EXAMPLE RUN TO DEMONSTRATE C.R.O.P.

INPUT DATA

O/ALL RECOV. DATA	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	TOT/AV
Pol % Cane	10.80	11.70	12.70	13.30	13.90	13.70	13.50	12.70	12.00	11.40	10.60	10.30	
Fibre % Cane	16.20	15.60	15.00	15.00	15.00	15.10	15.40	15.80	16.10	16.20	16.40	16.50	
Fib-in-bag % Cane	16.30	15.10	14.50	14.50	14.50	14.60	14.90	15.30	15.60	15.70	15.90	16.00	
Mixed juice pur	81.00	83.60	85.10	85.30	85.10	85.00	84.00	83.60	83.00	81.80	79,90	79.00	
R.S./Ash ratio	1.70	1.70	1.40	1.30	1.30	1.10	1.30	1.40	1.40	1.50	1.60	1.70	
Undetermd. loss	1.80	1.50	1.40	1.40	1.40	1.50	1.70	1.60	1.80	2.00	2.20	2.20	
Impur. Rec. ratio	0.81	0.81	0.81	0.81	0.81	0.81	0.81	0.81	0.81	0.81	0.81	0.81	
Pol/Sucr. ratio	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	
Filtercake loss	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	
CRE @ Ref. fib/hr	97.00	97.00	97.00	97.00	97.00	97.00	97.00	97.00	97.00	97.00	97.00	97.00	
TPD @ Ref. bx/hr	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	
Ref. tons fib/hr	34.00	34.00	34.00	34.00	34.00	34.00	34.00	34.00	34.00	34.00	34.00	34.00	
Ref. tons bx/hr	40.00	40.00	40.00	40.00	40.00	40.00	40.00	40.00	40.00	40.00	40.00	40.00	
Ref. tons ns/hr	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	
CRE Drop/fib/hr	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	
TPD Incr./bx/hr	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	
TPD Incr./ns/hr	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	

MONTHLY CONSTRAINTS

Max. tons fib/hr	42.00	42.00	42.00	42.00	42.00	42.00	42.00	42.00	42.00	42.00	42.00	42.00
Max. tons pol/hr	38.00	38.00	38.00	38.00	38.00	38.00	38.00	38.00	38.00	38.00	38.00	38.00
Max. tons bx/hr	43.00	43.00	43.00	43.00	43.00	43.00	43.00	43.00	43.00	43.00	43.00	43.00
Max. tons ns/hr	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
Max. ton cane/wk	42000.	42000.	42000.	42000.	42000.	42000.	42000.	42000.	42000.	42000.	42000.	42000.
Min. ton cane/wk	28000.	29000.	32000.	32000.	32000.	32000.	32000.	31000.	28000.	28000.	28000.	21000.
Hrs/wk operat'n	152.00	152.00	152.00	152.00	152.00	152.00	152.00	152.00	152.00	152.00	152.00	152.00
Crush time eff.	83.0	93.0	95.0	95.0	93.0	92.0	92.0	92.0	84.0	85.0	83.0	83.0

CONSTRAINTS APPLICABLE TO ENTIRE SEASON

Tons cane crushed for the season: Maximum 1 300 000 Minimum 1 300 000
Starting dates: Earliest 4.50 Latest 5.50
Finishing dates: Earliest 12.50 Latest 1.50 Note: 1st Jan = 1,0, 21st Dec = 12.67, etc.

DIVISION OF PROCEEDS DATA FOR THE SEASON

S.A.S.A. Variable cost requirements (Incl. Transport) for: Millers: R2,66 Growers: R3,88 (per ton cane)

Base cane tonnage: 1 300 000 tons % of Industry : 6,5%
Base sugar tonnage: 145 000 tons Average mill door price: R176,00 / ton sugar
Base sucrose tonnage: 170 000 tons Sucrose price in cane: R89,00 /ton sucrose

FIXED AND VARIABLE COSTS OF OPERATION

Description:	Fixed cost, Rand/season	Variable cost, Rand/ton cane:	Time cost, Rand/crush week:
Overheads	1,300,000.		
Depreciation	700,000.		
Wages	1,100,000.	0.00	4,000.
Rations	60,000.	0.00	0.
Stores	1,200,000.	0.06	1,000.
Transport	0.	1.30	0.
Other	400,000.	0.00	0.

Exhibit - 1: Input Data

* Tons fibre per hour, representing capacity limitations on the extraction plant (This is shown in Figure).
* Tons pol per hour in mixed juice, representing limitations on A-crystalliser and A-centrifugal capacity
* Tons brix per hour in mixed juice, representing capacity limitations in the front end of the boiling house.
* Tons non-sucrose per hour in mixed juice, representing capacity limitations in the back end of the boiling house

If the relationship between CRE and imbibition water % fibre were known with reasonable accuracy, an evaporator or boiler constraint on the tons/hour imbibition water could also have been included.

4.2. *Time utilisation of the mill:*

The mill will obviously not crush at any of the above hourly rates all week, because of the time loss due to the 14 to 24 hours scheduled maintenance shut-down each week and stoppages during the operational part of the week, caused by lack of cane, breakdowns, chokes, etc.

These considerations are taken into account, firstly by specifying the expected number of planned hours of crushing per week in each month (= 168 hours - hours of planned maintenance shut-down time), and the crushing time efficiency which, for these purposes, is defined as the percentage of the above planned operating hours that the mill is likely to be crushing, after taking into account the expected effect of the various types of stoppages.

Because the decision variable in the optimisation is expressed in terms of tons cane per week, the abovementioned tons/hour rate constraints must similarly be expressed. For example:

Max. tons cane/week, as constrained by brix in mixed juice throughput rate

$$= \text{Max. tons Bx/h in MJ} \times \text{Hours crush/week} \times \text{Crush time eff.} \times \frac{\text{MJ Purity}}{\text{Pol \% cane} \times \text{Extr.}}$$

Such calculations are automatically done by the program.

4.3. *Cane supply constraints:*

Besides the constraints on the processing capabilities of the mill, there might also be a minimum constraint, in that the growers would not be prepared to deliver less than a certain amount of cane per week. In order to reduce the length of the computer search and thus the running time for the program, it is in any case desirable to specify some realistic lower limit on weekly cane supplies instead of zero.

In addition to constraints on the weekly cane supplies, there are most likely to be constraints on the total amount of cane to be supplied over the season. Provision is made for an upper and a lower constraint, but if the total amount of cane to be crushed over the season is fixed, both constraints on the total season crush are given the same value.

A word of caution: One should be careful in assigning separate upper and lower limits to the total cane crushed for the season. Unless the drop in overall recovery with increased throughput rate is very severe, the program results will probably show that, for maximum profit, the total crush should equal the upper seasonal constraint. When viewing the season in isolation, as the program indeed does, this would be a perfectly correct conclusion, but the program does not take into account the possibility that the maximum seasonal cane

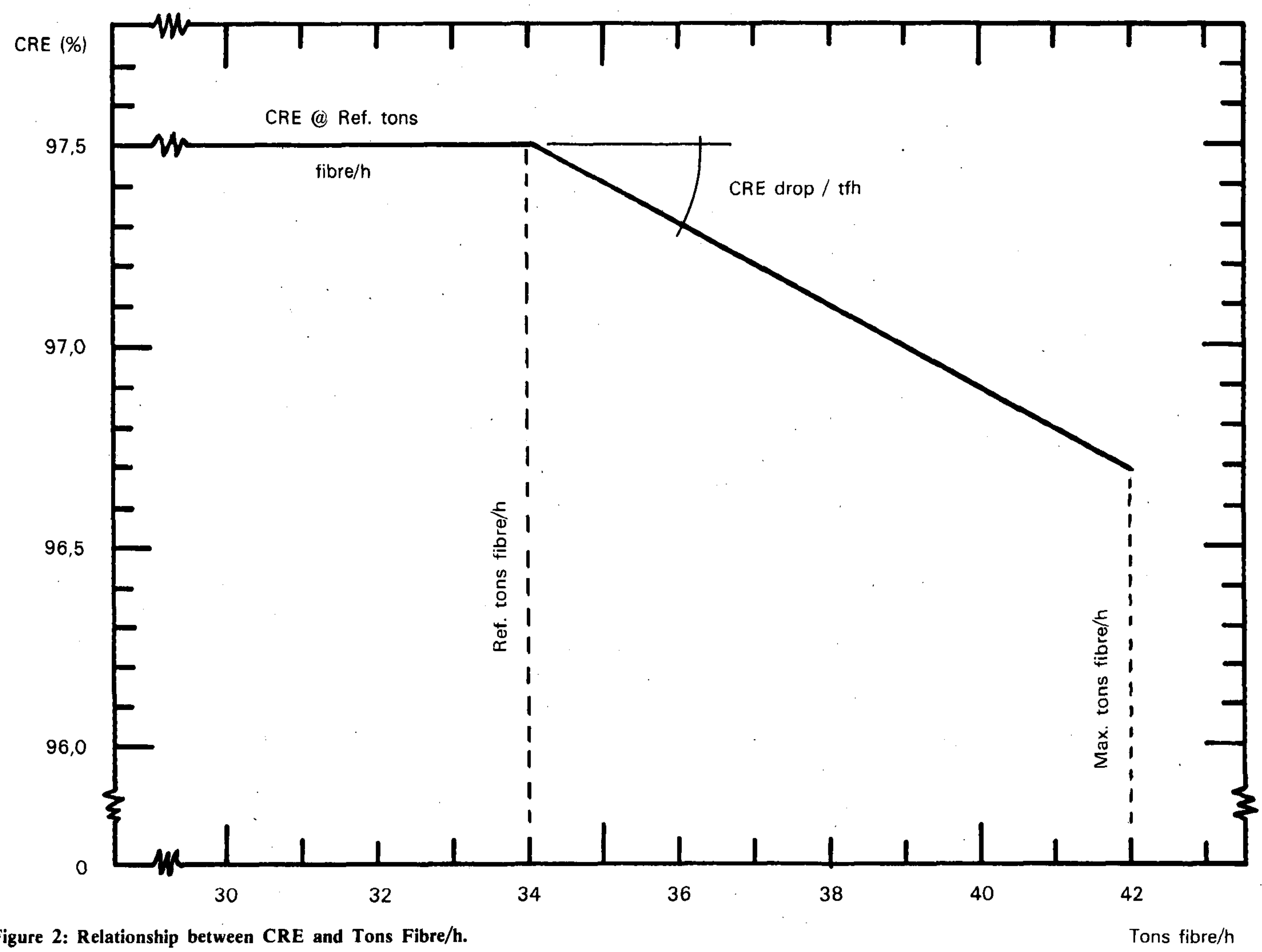

Figure 2: Relationship between CRE and Tons Fibre/h.

availability might have been achieved by over-cutting, at the cost of cane supplies for the following season. In most cases, it might therefore be safer to keep the total amount of cane to be crushed for the season at a fixed value.

4.4 *Starting and finishing dates of season:*

The program will choose the optimum starting and finishing dates for the season, within the bounds of any constraints which might be imposed, these being an earliest and a latest starting date, and an earliest and a latest finishing date. If it is desired to rigidly adhere to, say, a definite starting date, that date should then be used both as the earliest and the latest starting date constraint values.

Dates are given on the time scale of 1.00 representing 1st January, 6.50 representing 16th June, 12.97 representing 31st December, etc.

5. Costs

The costs shown in Exhibit 1 do not represent those of any particular mill, but are reasonably realistic.

Each type of cost incurred can have one or more of the following components:

5.1 *Fixed costs:*

These cost components are fixed for the entire 12-month year, irrespective of duration of season or amount of cane crushed. For their allocation per month, these costs are simply divided by 12. The values of fixed costs will not affect the optimisation result, but are provided to let the program calculate the correct final profit for the year.

5.2. *Costs/ton cane:*

These cost components are directly related to the amount of cane crushed, and are expressed as Rand/ton cane. Examples are cost of chemicals and growers' transport inflation. The level of these variable costs will only affect the final optimisation answer if the program is given some latitude between the maximum and minimum cane tonnage constraints for the season.

5.3. *Cost/week of crushing season:*

Certain cost components are directly dependent on the length of the crushing season, irrespective of the amount of cane crushed. Examples are: Wages of people who are employed only during the crushing season and are retrenched for the off-crop; lubricants which are changed or replenished on a daily or weekly basis; additional off-crop overtime costs necessitated by the longer crushing season and correspondingly shorter off-crop. In runs where the program is given latitude in choosing the starting and/or finishing date of the season, the level of this cost will affect the optimisation result.

Certain costs, such as stores, can be made up of all 3 of the above cost components.

6. Incomes

Due to the operation of the present Division of Proceeds System followed in South Africa, whereby the millers' cost requirements are proportionately shared out over the tonnages sugar produced, the value received by an individual mill for every *additional* ton of sugar produced will be less than the average Mill Door Price by a percentage approximately equal to the percentage of sugar industry production which that particular mill contributes.

If however such additional sugar production was achieved by harvesting and crushing additional cane, the drop in marginal income will not be quite as severe, because of the official industry variable costs of growing and milling being added to the total industrial requirements.

To cater for these Division of Proceeds effects, the necessary information has to be supplied as part of the input.

7. Output Results

Exhibit 2 shows the results from the optimisation.

The number of weeks crushed each month is stated, from which the starting and finishing dates can be calculated, and the crushing rate in terms of tons cane per week. Also given are the hourly cane, fibre, pol, brix and non-sucrose rates. Where an active upper constraint is encountered, such as brix rate in September or fibre rate in October, this is indicated by "*", and for an active lower constraint such as tons cane/week in December by "@". In a month such as May where neither upper nor lower constraint is encountered, the program has chosen the crushing rate purely on the basis of balancing the benefit of a shorter season against the poorer recoveries of faster crushing.

The pol and sugar tonnages are listed month-by-month and for the season, and the figures for extraction, boiling house recovery and overall recovery are given month-by-month as well as a weighted mean for the season.

The income from sugar sold, cost of sucrose purchased and milling margin are calculated by taking into account the operation of the Division of Proceeds system.

The monthly and annual costs of each department are given, made up of the fixed, per ton cane and per crushing week components where applicable. The milling margin less total costs gives the profit, which will be the maximum under the given conditions.

The computer used is an ICL 2904. The program requires 24K words of storage, with a running time of about 2 minutes.

8. Uses of the crushing rate optimisation program

Besides the most obvious use of the program, namely to specify the optimum crushing rate policy for the coming season within the constraints of mill capacity, cane availability and allowable range of starting and finishing dates for the season, this crushing rate optimisation program can also be used for the following purposes:

8.1. *Calculating the effect of taking in more cane:*

Taking in additional cane should increase the sugar production from the mill and thus its revenue, but offset against this is the lower sugar produced/ton cane through higher loadings of the equipment and/or a lengthened crushing season. CROP will calculate to what extent the crushing rate has to be increased and season lengthened to accommodate the additional cane with maximum profit.

8.2 *Determine the merits of additional capital investment:*

Capital is often invested in additional equipment to provide higher overall recoveries, or else to increase the maximum possible throughput and thus reduce the length of crushing season. CROP will calculate the new recommended crushing rate policy as well as the expected profit. Using discounted cash flow techniques, the additional profit over the years should then be compared with the capital cost of the necessary equipment to decide whether the investment is worthwhile.

8.3. *Answering of "What - if" questions:*

Instead of specifying an optimum crushing policy, the program can equally well calculate the effects of following a given policy of starting crushing on a certain date, crushing at a specified rate each month, and ending the season on a specific date. The program will calculate the monthly extraction, boiling house recovery, sugar production and sucrose purchases, as well as the income and expenditure each month and for the season. This is easily achieved by setting the earliest and the

OPTIMIZATION OF MILL CRUSHING PROGRAM **DATE: 13/04/78** **TIME: 10/46/00**

MILL: EXAMPLE MILL **SEASON: 1978/9** **RUN No.: 1**

OBJECT: EXAMPLE RUN TO DEMONSTRATE C.R.O.P.

OUTPUT RESULTS

PERFORMANCES	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	TOT/AV
Given:													
Hrs/wk operat'n	152.	152.	152.	152.	152.	152.	152.	152.	152.	152.	152.	152.	
Crush time eff.	83.0	93.0	95.0	95.0	93.0	92.0	92.0	92.0	84.0	85.0	83.0	83.0	
From optimization:													
Wks. crushed/mo.	0.00	4,33	4.33	4.33	4.33	4.33	4.33	4.33	4.33	1.24	0.00	0.00	35.90
Tons cane/week	0	32000	40000	40000	38000	39000	38000	37000	28000@	28000@	00	00	
Tons cane/month	0	138667	173333	173333	164667	169000	164667	160333	121333	34667	0	0	1300000*
Hourly rates:													
Tons cane/hour	0.0	226.4	277.0	277.0	268.8	278.9	271.7	264.6	219.3	216.7	0.0	0.0	
Tons fibre/hour	0.0	35.3	41.6*	41.6*	40.3	42.1	41.8*	41.8*	35.3	35.1	0.00	0.00	
Tons pol/hour	13.0	25.6	33.9	35.6	36.2	36.9	35.4	32.3	25.4	23.9	0.00	0.00	
Tons brix/hour	0.0	30.7	39.9	41.7	42.5*	43.4*	42.1	38.7	30.6	29.2	0.00	0.00	
Tons N.suc/hour	0.0	5.0	5.9	6.1	6.3	6.5	6.7	6.3	5.2	5.3	0.00	0.00	
Recoveries:													
Pol % cane	10.80	11.70	12.70	13.30	13.90	13.70	13.50	12.70	12.00	11.64	0.00	0.00	12.96
Tons cane pol/mo	0	16224	22013	23053	22889	23153	22230	20362	14560	4034	0	0	168518
Extraction	96.44	96.75	96.46	96.55	96.75	96.54	96.45	96.20	96.67	96.56	0.00	0.00	96.54
B/house recov.	88.82	89.54	90.08	89.92	89.76	89.03	88.72	88.89	88.27	87.89	0.00	0.00	89.28
O/all recovery	89.3	86.6	86.9	86.8	86.9	85.9	85.6	85.5	85.3	84.9	0.0	0.0	86.2
Tons sugar prod	0	14054	19126	20016	19878	19899	19021	17411	12424	3423	0	0	145252
Values (in R1 000):													
Sugar sold	0	2473	3366	3522	3498	3502	3347	3064	2186	602	0	0	25560
Cane purchased	0	1445	1960	2053	2038	2062	1980	1813	1297	359	0	0	15007
Milling Margin	0	1028	1406	1469	1460	1440	1367	1251	889	243	0	0	10553
Costs:													
Overheads	108	108	108	108	108	108	108	108	108	108	108	108	1296
Depreciation	58	58	58	58	58	58	58	58	58	58	58	58	696
Wages	92	109	109	109	109	109	109	109	109	97	92	92	1245
Rations	5	5	5	5	5	5	5	5	5	5	5	5	60
Stores	100	113	115	115	114	114	114	114	112	103	100	100	1314
Transport	0	180	225	225	214	220	214	208	158	45	0	0	1689
Other costs	33	33	33	33	33	33	33	33	33	33	33	33	396
Total costs	396	606	653	653	641	647	641	635	583	449	396	396	6696
Profit	-396	422	753	816	819	793	726	616	306	-206	-396	-396	3857

* Signifies operative upper constraint on throughput
@ Signifies operative lower constraint on throughput

Exhibit - 2: Output Results

latest starting date both equal to the specified starting date, and likewise for the finishing dates. For each month, the maximum and the minimum cane supply rates are both set equal to the specified cane supply rate for that month.

For such runs the computer time required is considerably less, because no search has to be made.

9. Concluding remarks

The potential user of such an optimisation program might have misgivings, because of the need to provide values for the constraints on the various throughput rates, as well as a quantification of how the CRE and the TPD will be affected by throughput rate. Because these values are generally not known accurately, and would in many cases be a bit of an inspired guess, it might be argued that there is no point in using such an optimisation program. However, the alternative is to let the crushing program for the season be one great thumb-suck, which not only implies the same guestimates of constraints and drops in recoveries, but any errors are greatly compounded by mentally attempting to relate such estimates to profits. On the other hand, for what the estimated constraints and recovery relationships are worth, the answer provided by CROP will be correct.

10. Acknowledgements

The contribution of Ian Smith in deriving the expression for overall recovery in terms of the laboratory analyses and performance yardsticks is acknowledged.

11. REFERENCES

1. Nemhauser, G. L. (1966) - Introduction to Dynamic Programming. Wiley, 14-84.
2. Smith I.A. (1976) - Differences in adjustment for cane quality between three factory performance yardsticks. SASTA Proc. 50 231-236.

APPENDIX 1

Derivation of Expression for Boiling House Recovery

Boiling house recovery is given as

$$BHR = 100 - L_c - L_u - L_m$$

where L_c = Pol in filter cake % Pol in MJ
L_u = Pol in undetermined loss % Vol. in MJ
L_m = Pol in molasses % Pol in MJ

The pol lost in molasses can be expressed as

$$\frac{\text{Pol in molasses}}{\text{Pol in MJ}} = \frac{\text{Sucr. in mol.}}{\text{NS in mol.}} \times \frac{\text{Pol in mol.}}{\text{Sucr. in mol}} \times \frac{\text{NS in mol.}}{\text{NP in MJ}} \times \frac{\text{NP in MJ}}{\text{Pol in MJ}}$$

$$\text{or } L_m = 100 \left(\frac{M}{100\text{-}M}\right) \ HR \ \left(\frac{100 - J}{J}\right)$$

where M = True purity of molasses
H = Pol/sucrose ratio in molasses
R = Non-sucrose in molasses/Non-pol in MJ
J = Mixed Juice purity

But M = T + Target purity
= T + 39.94 - 19.6 log G

where T = Target purity difference of final molasses
G = Reducing sugar/Ash ratio in molasses

Therefore,

$$BHR = 100 - L_c - L_u - 100\ HR \left(\frac{T + 39.94 - 19.6 \log G}{100 - (T + 39.94 - 19.6 \log G)}\right)\left(\frac{100\text{-}J}{J}\right)$$