AN APPLICATION OF THE FRACTIONAL FACTORIAL EXPERIMENT TO CONTINUOUS PAN BOILING

By G.A. MATTHESIUS
Sugar Milling Research Institute
and
W.S. GRAHAM and J.V. PILLAY
The Tongaat Group Ltd

Abstract

The underlying theory for a fractional factorial experiment has been outlined and its application demonstrated for the establishment of the operating conditions giving maximum throughput for a continuous pan boiling C-massecuite.

Introduction

The importance of good planning for any experimental investigations is generally acknowledged (Federer[1], Raghavarao[2]). The most important aspect is the selection of the right procedure or technique to obtain the required information. Sometimes fundamental statistical rules and assumptions are disregarded (Wine[3]) so that wrong conclusions are then drawn by the interpretation of the statistical results.

The efficient planning and proper design of experiments is discussed in cited literature and these references may be used for further study if desired. A short introduction to this subject was given by Lionnet & Baker[4] and their method of evaluating and interpreting results obtained from a complete factorial experiment was used as the basis of this paper. The object of this paper is twofold.

1. An introduction to the mathematical techniques so that the advantages and disadvantages of a fractional factorial experiment are highlighted. Further it should encourage the use of planned and statistically based investigations.
2. By means of a practical problem, i.e. maximisation of the throughput of the continuous vacuum pan at Tongaat sugar factory, the application and interpretation of these theoretical considerations is demonstrated.

All statistical calculations were carried out on a Wang 2200 computer using BASIC programmes either developed or modified at the SMRI.

The fractional factorial experiment

When the experimenter wants to investigate the influence of a certain number, m, of factors on a response, he may find that the number of tests required may become prohibitive. The number of combinations can be cut down considerably by investigating each factor at two levels only. As was shown by Mendenhall[5], a two level factorial provides sufficient information to fit a plane to a response surface and also permits an estimation of the effects of interactions between factors although the planar fit is really inadequate for such evaluation. Myers[6] proved that a first order model will hold true for any part of a high order surface, if the experimental range is small enough. Only the number of factors then determines the necessary factor combinations (2^m) of a complete factorial experiment and these combinations form an Abelian group (Fisher[7]). However, the required number of tests may still be too large. It is then often possible to divide a full factorial into two, four or more blocks in such a way that primary factors and interactions are confounded with interactions of only minor importance to the experiment.

The great advantage of limiting the size of a block lies in the fact that in this way the experimental conditions may be made much more homogeneous than for a larger size. Thus the required information will be obtained in a shorter time at a lower price if a one block experiment or fractional factorial experiment is carried out.

The theory underlying the construction of a fractional factorial experiment is beyond the scope of this paper. The basic principle can be explained by means of a 2^2 experiment. Denoting the two factors A and B, the lower level of each by – and the upper level by +, the four possible treatment combinations for a complete factorial experiment are given in Table 1.

TABLE 1

2^2 Factorial experiment

Test No.	Treatment combination	Factor level		Interaction (AB)	Yield
		A	B		
1	(I)	–	–	+	y_1
2	a	+	–	–	y_2
3	b	–	+	–	y_3
4	ab	+	+	+	y_4

If the four tests are carried out under conditions corresponding to the factor level notations, a statistical evaluation of the results (yield) will determine whether a level change in A or B influences the yield significantly. It is also possible to estimate the effect of the interaction between A and B on the yield.

Assuming the two factors A and B do not interact, the comparison represented by AB will be zero, apart from experimental error. It is then possible to utilise AB to measure the effect of a third factor, C, if it can be assumed that the new factor will not interact with the other factors. When C is equated with AB in Table 1, a design for three factors in four observations or a $\frac{1}{2} * 2^3$ factorial experiment (fractional factorial experiment) is obtained. The treatment combinations are given in Table 2.

TABLE 2

$\frac{1}{2} * 2^3$ Factorial experiment

Test No.	Treatment combination	Factor level			Yield
		A	B	C	
1	c	–	–	+	y_1
2	a	+	–	–	y_2
3	b	–	+	–	y_3
4	abc	+	+	+	y_4

The evaluation of this fractional factorial experiment is complicated because it is not possible to determine the pure effects of A, B and C, but only the confounded effects of A + BC, B + AC and C + AB. It is therefore evident that the abovementioned assumption has to be valid to obtain meaningful statistical results.

The results are used to establish a linear equation which substitutes the actual response surface for the investigated area. The general formula for a 2^2 experiment is:

$$y = \underbrace{\beta_0 + \beta_1 x_1 + \beta_2 x_2}_{\text{main effects}} + \underbrace{\beta_3 x_1 x_2}_{\text{two way interaction}} + \underbrace{e}_{\text{experimental error}}$$

and therefore a ½*2³ experiment is described by

$$y = ß_0 + ß_1 x_1 + ß_2 x_2 + ß_3 x_3 + e$$

When applying this method to an actual problem the number of independent variables in this type of equation will finally be reduced to the number of factors and interactions which have a statistically significant influence on the yield. A multi-linear regression analysis determines the slope-values for the independent variables and it is then possible, by calculating the steepest ascent to an extremum, to decide on the best settings for the important variables. If the experimenter is limited to a certain practical range for his investigations, a determination of the most favourable treatment combinations for the yield within the experimental area can be achieved.

The principle of substituting minor important interactions by new factors is used for any fractional factorial experiment. The practical problem is to find out whether a particular design will give the required information. It is therefore necessary to determine, for various substitutions, the confoundings between main effects/interactions and interactions/interactions to attempt to obtain a suitable experimental design. These investigations have to be carried out for each problem individually because it is quite possible that no fractional design will give revealing answers.

Apart from this restricted applicability and the problems concerning the estimation of the experimental error (Davies[8]), two major disadvantages of a fractional factorial experiment can be pointed out.

1. When a new factor replaces an interaction, it must be assumed that the influence of this interaction on the yield is negligible and is set to zero.
2. The introduction of a new factor is permissible only if there are no interactions between the old factors and the new factor – a requirement, which is difficult to verify in practice.

An application of a fractional factorial experiment

A 64 m³ continuous vacuum pan was put into operation during the 1976/77 season at Tongaat sugar factory. The technical details, operating experiences and the performance of this pan boiling low grade massecuite in its first season is reported by Graham & Radford[9]. The throughput of C-massecuite appeared to be less than would be expected from data reported by Langreney[10] and Broadfoot & Allen[11] for continuous pans. Hence it was desired to optimise the throughput by an evaluation of the effects of certain controllable variables.

Factors such as C-massecuite purity were not considered to be acceptable variables. It is well known that by raising the C-massecuite purity, other things being unaltered, the output of the pan would be increased but this would be accompanied by a rise in final molasses purity. Six acceptable factors were considered to be significant and these are listed in Table 3 together with their minimum and maximum values. The explanation of the low and high brix profile is illustrated graphically in Fig. 1, which shows the progressive increase in massecuite brix through the pan. As far as possible the purity and brix of the seed, feed molasses and C-massecuite were kept constant as these would be expected to affect the results significantly.

Design and tests

Since the investigation involved a number of factors which were likely to interact, a factorial experiment for the 6 factors, each at two selected levels, would require 2^6 or 64 tests. In practice, it would not be possible to complete these runs without having an uncontrolled time influence on the yield. It was therefore decided that 16 runs plus 7 replicates for the estimation of the experimental error would be practicable. Preliminary tests indicated that a change in the factor "vacuum" was not acceptable in practice. Hence, this variable was eliminated and kept constant at 9 kPa abs. The full working range for each factor was used in setting the minimum and maximum levels in order to minimise the contribution of experimental error.

TABLE 3

Variables and interactions for the fractional factorial experiment

Factor	Abbreviation	Practical working range
Seed rate	A	3,11 ... 3;67 t/h
Brix profile	B	Low ... High
Steam pressure	C	105 ... 140 kPa abs.
Massecuite level*	D	100 ... 400 mm
Jigger pressure	E	70 ... 100 kPa abs.
Vacuum	–	9 ... 12 kPa abs.
Seed rate/Brix profile	AB	
Seed rate/Steam pressure	AC	
Seed rate/Massecuite level	AD	
Brix profile/Steam pressure	BC	
Massecuite level/Jigger pressure	DE	

* These values relate to height above minimum weir setting. The corresponding massecuite levels above the top of the calandria are 500mm and 800mm.

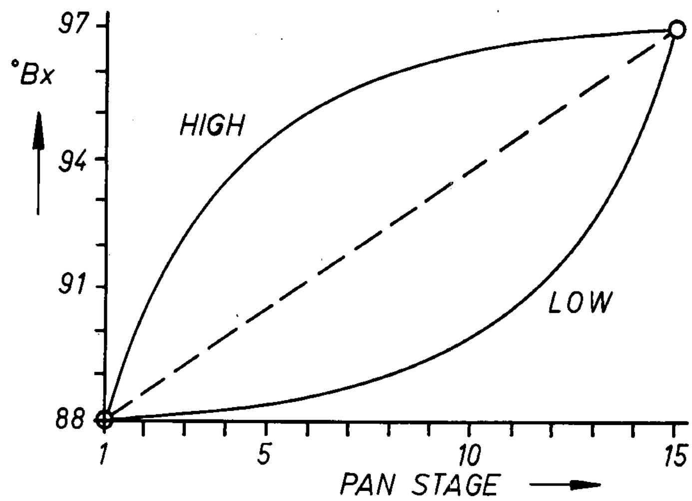

Figure 1: Brix profiles along the stages of the continuous vacuum pan.

As only 16 tests were planned, the first 4 factors of Table 3 were used to design a complete 2^4 factorial experiment, to which the 5th factor "jigger press." was added by substituting it for one interaction. A computer program considered all possible substitutions and determined the corresponding confoundings between factor/interaction and interaction/interaction. According to these investigations the ½*2⁵ experiment which will give most of the required information was designed by equating the 4th order interaction with the new factor. Table 4 shows that apart from the last effect in Table 3 (DE is zero because E should not interact with any other factor) all interesting factors can be evaluated because they are confounded with higher order interactions which are likely to be zero.

The final design for the ½*2⁵ factorial experiment and its treatment combinations for the 16 tests is elaborated in Table 5. The tests were carried out in random order and the corresponding throughputs are listed in the last column of Table 5. The throughput values were obtained from measurement of the change in level of a strike receiver over a 30 minute period. The average of the individual results obtained during an 8 hour test run gave the volume of massecuite produced. This was converted to tons using a density factor of 1,5.

All conditions as well as the throughput measurements were checked during the tests on an hourly basis to obtain information on the variability of the conditions. Table 6 shows average values, $\bar{x}$, for the individual experiment conditions and the

1,96*s – intervals which contain statistically 95% of the measured values (Sachs[12]). As indicated by this table the variations for the "constant" factors were considerable but they represent actual factory conditions under which the pan has to work.

TABLE 4

Fractional factorial experiment: half – replicates (16 tests)

E Replaces ABCD Defining contrasts: I and ABCDE	
EFFECTS	CONFOUNDED WITH
A	BCDE
B	ACDE
C	ABDE
D	ABCE
E	ABCD
AB	CDE
AC	BDE
AD	BCE
AE	BCD
BC	ADE
BD	ACE
BE	ACD
CD	ABE
CE	ABD
DE	ABC

TABLE 5

½*2[5] Fractional factorial design (16 tests, each factor at 2 levels)

Test No.	Treatment combination	Level of factor					Response variable
		Seed	Brix	Steam	Level	Jigger	Throughput
		A	B	C	D	E=+ABCD	(t/h)
1	e	–	–	–	–	+	8,87
2	a	+	–	–	–	–	7,76
3	b	–	+	–	–	–	9,43
4	abe	+	+	–	–	+	8,65
5	c	–	–	+	–	–	12,13
6	ace	+	–	+	–	+	12,29
7	bce	–	+	+	–	+	12,62
8	abc	+	+	+	–	–	11,85
9	d	–	–	–	+	–	6,95
10	ade	+	–	–	+	+	7,89
11	bde	–	+	–	+	+	8,29
12	abd	+	+	–	+	–	7,58
13	cde	–	–	+	+	+	9,00
14	acd	+	–	+	+	–	9,87
15	bcd	–	+	+	+	–	9,85
16	abcde	+	+	+	+	+	11,11

Analysis of results

(Davies[8], Davies & Goldsmith [13])

The experimental error (standard deviation) of the investigations was estimated by means of a statistical analysis on throughput measurements for 8 runs on the randomly selected treatment combination (b). It was found to be:-

$$s_{experiment} = \left(\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \right)^{½} = 0{,}334$$

The experimental error of a single effect was calculated as follows:-

$$s_{effect} = \left(s^2_{experiment} * N/ (N/2)^2 \right)^{½} = 0{,}167$$

The statistical evaluation of the results obtained by the ½*2[5] fractional factorial experiment was carried out with a computer program which determined the effects of the factors and interactions on the throughput. To find out whether individual effects had a significant influence on the throughput, each effect was tested on the hypothesis

$$H_0 : EFFECT = 0$$

by calculating the 95% confidence interval (CI).

$$CI = EFFECT \pm t_{n-1,a} * s_{effect} = EFFECT \pm 0{,}394$$

$$t_{n-1,a} = 2{,}365$$

$$a = 5\%$$

TABLE 6

Typical values of experimental conditions

Condition		$\bar{x}$	$\bar{x} \pm 1{,}96*s$ ++
Steam pressure (KPa abs)	High	139,9	139,2 - 140,5
	Low	105,1	104,0 - 106,2
Jigger pressure (KPa abs)......	High	100,0	99,0 - 101,0
	Low	70,0	69,0 - 71,0
Brix profile	High	above	straight line
	Low	below	straight line
Massecuite level (mm)	High	400	398 - 402
	Low	100	98 - 102
Seed rate (t/h)	High	3,67	3,42 - 3,92
	Low	3,11	2,87 - 3,34
Vacuum (KPa abs)...........		8,8	const.
B-molasses brix		73,7	69,1 - 78,3
B-molasses purity		46,9	39,8 - 54,1
Seed brix		87,6	84,7 - 90,4
Seed purity		64,8	54,0 - 75,6
C-massecuite brix		96,1	95,4 - 96,8
C-massecuite purity		53,5	49,6 - 57,4

++ **Calculated range which does not necessarily reflect the actual data variations.**

If the confidence interval did not include the zero the hypothesis H_0 would be rejected, indicating that this effect had a significant influence on the throughput. The evaluation for the experiment is given in Table 7. An increase in factor level for "Brix profile, Steam press, Jigger press" and a decrease for "Massecuite level" gave an increase in throughput. When the influence of the significant interactions "Seed rate/Steam press, Seed rate/Massecuite level and Steam press/Massecuite level" was examined individually, it was concluded by indirect inference that the factor "Seed rate" had a negligible effect on the throughput, whereas a positive influence was anticipated.

A multi-linear regression analysis was carried out by means of a computer program to establish a mathematical relationship between throughput and operating conditions. The seven factors, underlined in Table 7, were used as independent variables for this analysis in the course of which the interactions were treated as being linear (Mendenhall[5]). Using the factor levels applied in the experiment with the corresponding throughput values the equation obtained was:-

TP = – 4,35 + 5,78E - 2 (BP) + 0,15 (SP) – 1,03E - 2 (ML) + 1,38E - 2 (JP) + 5,8E - 3 (ML*SR) – 1,2E - 4 (SP*ML) – 1,1E - 2 (SP*SR); r = 0,98

By varying the operating conditions within given practical ranges the maximum yield for throughput was calculated for the settings shown in Table 8. The theoretical value for throughput is in good agreement with the measured value. Further test runs were carried out to confirm this result and they yielded slightly lower throughput values (about 12,5 t/h). The difference may be explained as the result of variation in juice quality (nature of impurities) with time during the crushing season. Such a change is likely to influence the pan performance considerably.

TABLE 7
Confidence interval and 95% significance for factors

Factor	Confidence interval		Significance
SEED	-.412455	.377455	NOT SIGNIFICANT
BRIX	.182545	.972455	SIGNIFICANT
STEAM	2.517545	3.307455	SIGNIFICANT
LEVEL	-2.027455	-1.237545	SIGNIFICANT
JIGGER	1.75450000E-02	.807455	SIGNIFICANT
SEED/BRIX	-.627455	.162455	NOT SIGNIFICANT
SEED/STEAM	2.54500000E-03	.792455	SIGNIFICANT
SEED/LEVEL	.212545	1.002455	SIGNIFICANT
SEED/JIGGER	-8.74550000E-02	.702455	NOT SIGNIFICANT
BRIX/STEAM	-.437455	.352455	NOT SIGNIFICANT
BRIX/LEVEL	-.192455	.597455	NOT SIGNIFICANT
BRIX/JIGGER	-.317455	.472455	NOT SIGNIFICANT
STEAM/LEVEL	-1.027455	-.237545	SIGNIFICANT
STEAM/JIGGER	-.477455	.312455	NOT SIGNIFICANT
LEVEL/JIGGER	-.297455	.492455	NOT SIGNIFICANT

TABLE 8
Operating conditions for maximum throughput

Steam pressure	140 kPa abs.
Jigger pressure	100 kPa abs.
Massecuite level	100 mm
Brix profile	high
Seed rate	3.11 t/h
Throughput (calculated)	12,99 t/h
(measured)	12,62 t/h

Conclusion

The use of a fractional factorial experiment to establish the importance of certain specified variables on the capacity of a continuous pan boiling C-massecuite at Tongaat sugar factory was described. The results obtained using the technique were largely satisfactory.

Acknowledgements

The authors thanks are due to Dr. A. I. Dale (Dept. Mathematical Statistics, University of Natal, Durban) for advice on the statistical evaluations.

List of Symbols

e	experimental error
m	number of variables or factors
n	number of replicate runs
r	regression coefficient
s	standard deviation
t	value from t-table
x	variable
$\bar{x}$	average for variable
α	significance level
ß	coefficient
N	number of combinations in experimental design
BP	brix profile (low ≙ O, high ≙ 10)
JP	jigger pressure
ML	massecuite level
SP	steam pressure
SR	seed rate
TP	throughput

REFERENCES

1. Federer, W. T. (1963). Experimental design. MacMillan, New York. 544.
2. Raghavarao, D. (1971). Constructions and combinatorial problems in design of experiments. Wiley & Sons, New York. 386.
3. Wine, R. L. (1964). Statistics for scientists and engineers. Prentice-Hall, New York. 671.
4. Lionnet, G. R. E. and Baker, S. M. (1977). Experiment planning and the use of factorial designs. SASTA Proc **51**, 125-128.
5. Mendenhall, W. (1968). The design and analysis of experiments. Wadsworth Publishing Co., Belmont, Calif. 465.
6. Myers, R. H. (1971). Response surface methodology. Allyn & Bacon, Boston. 246.
7. Fisher, R. A. (1950). The theory of confounding in factorial experiments in relation to the theory of groups – In: Contributions to mathematical statistics. John Wiley & Sons, New York. 341-353.
8. Davies, O. L. (1956). The design and analysis of industrial experiments. Oliver and Boyd, London. 636.
9. Graham, W. S. and Radford, D. J. (1977). A preliminary report on a continuous C pan. SASTA Proc **51**, 107-111.
10. Langreney, F. (1976). Preliminary results with the continuous Langreney vacuum pan. Z. Zuckerind. **101**, 772-776.
11. Broadfoot, R. and Allen, J. R. (in press). Continuous low grade massecuite boiling studies, ISSCT Proc **16**, (Brasil).
12. Sachs, L. (1970). Statistische Methoden. Springer, Heidelberg. 105.
13. Davies, O. L. and Goldsmith, P. L. (1977). Statistical methods in research and production. Longman, London. 478.