NATAL SUGAR MILL RESULTS.

EXAMINED IN THE LIGHT OF NOËL DEERR FORMULÆ.

By R. M. BECHARD.

Objects of Paper.

Of the many methods of analysis of sugar factory results, one of the most promising is that first proposed by Noël Deerr.[1]

The object of this paper is to examine the industrial results obtained in Natal during the past thirteen years, so as to establish in what way the practice in this country has conformed to the underlying theory of Deerr, which can be stated as follows :—

1. The average units of a calculated juice called primary juice lost for each unit of fibre in the cane, is a measure of the efficiency of extraction, and the corollary : that this efficiency should not be affected by a mere change in the fibre percentage.
2. That the average units of sucrose lost per unit of impurity in the mixed juices (called the virtual molasses coefficient) is a measure of the efficiency of the boiling, and its corollary : that this efficiency should not be affected by a mere change in the purity degree of the juice.

If these coefficients were true measures of efficiency, then at equal efficiency, the extraction obtainable with any other fibre percentage, and the recovery accruing from any other degree of purity, can be readily calculated.

Calculation of Coefficients.

These coefficients can, of course, be obtained from quantity measurements, but when these are not available, Deerr gave us the mathematical formulæ for obtaining them.

(1) $$V = \frac{(1-e)\,(1-f)}{f},$$

where V is the unit loss in primary juice,
e is the extraction expressed decimally,
f is the fibre per cent. cane expressed decimally.

(2) $$Mv = \frac{(1-e)\,j}{(1-e)\,j + (1-)j},$$

where Mv is the virtual molasses coefficent,
e is the B H Recovery expressed decimally,
j is the purity of the mixed juice expressed decimally.

There is no question that these formulæ for given values of "f" and "j" do yield the coefficients required.

Reduced Extraction and Recovery.

Deerr further proposed that an extraction calculated from a standard fibre (preferably 12.5 per cent.) and the mean unit loss, and a recovery calculated from a standard purity (preferably 85) and the mean virtual molasses, be the universal standard of comparison for efficiency. These values he called Reduced Extraction and Reduced Recovery.

This further suggestion, however pleasing for the purpose of showing an increased efficiency, when reporting results obtained from fibres higher than standard, and purities lower than standard, introduces the following assumptions :—

1. That the introduction of an arbitrary standard will not vitiate the interpretation of the results.
2. That a mere change in fibre per cent. or of degree of purity will not cause a change in the coefficients of unit loss.

Choice of Standard.

It can readily be shown that the first condition can hardly be met.

The inverted formula for obtaining the reduced extraction reduces to a formula of the type :

$$e = \frac{X - V}{X}$$

and that for reduced recovery is

$$e = \frac{X - m}{(1-m)}.$$

Both these formulæ will react strongly to a varying X, the net results being almost entirely dependent on the choice of X.

Comparison by Formulæ.

A simple example will illustrate the danger of a choice of standard.

Two factories, A and B, have the same fibre, say 20 per cent. They have respective extractions of 90 and 92.

Then clearly the difference of extraction efficiency is 2.

Their unit loss in primary juice will, however, be respectively 0.40 and 0.32, which on a standard of 12.5 per cent. fibre will yield reduced extractions of 94.29 and 95.43, a difference in efficiency of only 1.14.

The same can be shown for recovery.

Two factories having the same mixed juice purity, say 78, have respectively recoveries of 78 and 80.

Clearly again the difference in recovery efficiency is 2.

Their virtual molasses coefficients, however, are respectively 0.4351 and 0.4118, which on a standard purity of 85 will yield reduced recoveries of 86.24 and 87.49, a difference in efficiency of only 1.25.

Quality Value by Formulæ.

The same bewildering differences will accrue if we wished to separate from actual results the gains due to efficiency from that due to so-called quality factors.

Thus if we used standard fibres of 12.5 and 20 per cent. to analyse the results obtained during different periods when varying unit losses were obtained, we might be led to the following table of values :—

Fibre.	Actual extraction.	Unit loss.	Reduced extractions at 12.5% fibre.	Reduced extractions at 20.0% fibre.
14.748	92.195	0.4512	93.554	88.720
15.657	90.045	0.5365	92.336	86.588
0.909	2.150	—	1.218	2.132

From which we might be tempted to conclude that, out of a total gain of extraction of 2.150 either 2.132 or 1.218 was due to efficiency gain, and that a decrease of 0.909 in fibre resulted in a gain of extraction, which could be either 0.932 or 0.018.

These conclusions are too indefinite to be of any value.

Similarly, if we use standard purities of 80 and 85 and analyse the results obtained during periods when different virtual molasses were registered, we may construct the following table :—

Purity.	Actual BH Recovery.	Virtual Molasses.	Reduced B.H. recoveries at 80.0 Purity.	Reduced B.H. recoveries 85.0 Purity.
86.196	88.603	0.41578	82.208	87.441
85.276	85.015	0.46464	78.302	84.684
0.920	3.588		3.906	2.757

When we might again be tempted to conclude that, out of a total gain of recovery of 3.588, either 3.906 or 2.757 was due to efficiency gain, and that an increase of purity of 0.920 resulted in either a loss of recovery of 0.318 or a gain of 0.831.

These conclusions are therefore valueless.

The above examples have served to show that the method of reduced extraction and recovery is dangerous—it is not even necessary. Logically, always presuming that the second assumption was correct, we are only entitled to seek an answer to the following types of question :—

An extraction of 92.195 was attained when the fibre was 14.748, the coefficient of unit loss being 0.4512. What would have been the extraction had the fibre during the same period been 15.657 and assuming that the unit loss was not affected by the change in fibre ?

Or—A recovery of 88.603 was attained when the purity was 86.196, the coefficient of virtual molasses being 0.41578. What would have been the recovery during the same season had the purity been 85.276 and assuming that a change of purity did not cause a change in the virtual molasses ?

Validity of Results.

Before, however, we can attach importance to the index of efficiency indicated by the coefficients of unit loss, or attempt to value a change in fibre or purity for extraction or recovery purposes, we must investigate the second assumption. For should we find that a change in fibre or of purity is generally associated with a change of unit loss, the formulæ as they stand would be of little value.

This investigation is only possible by means of an examination of industrial results.

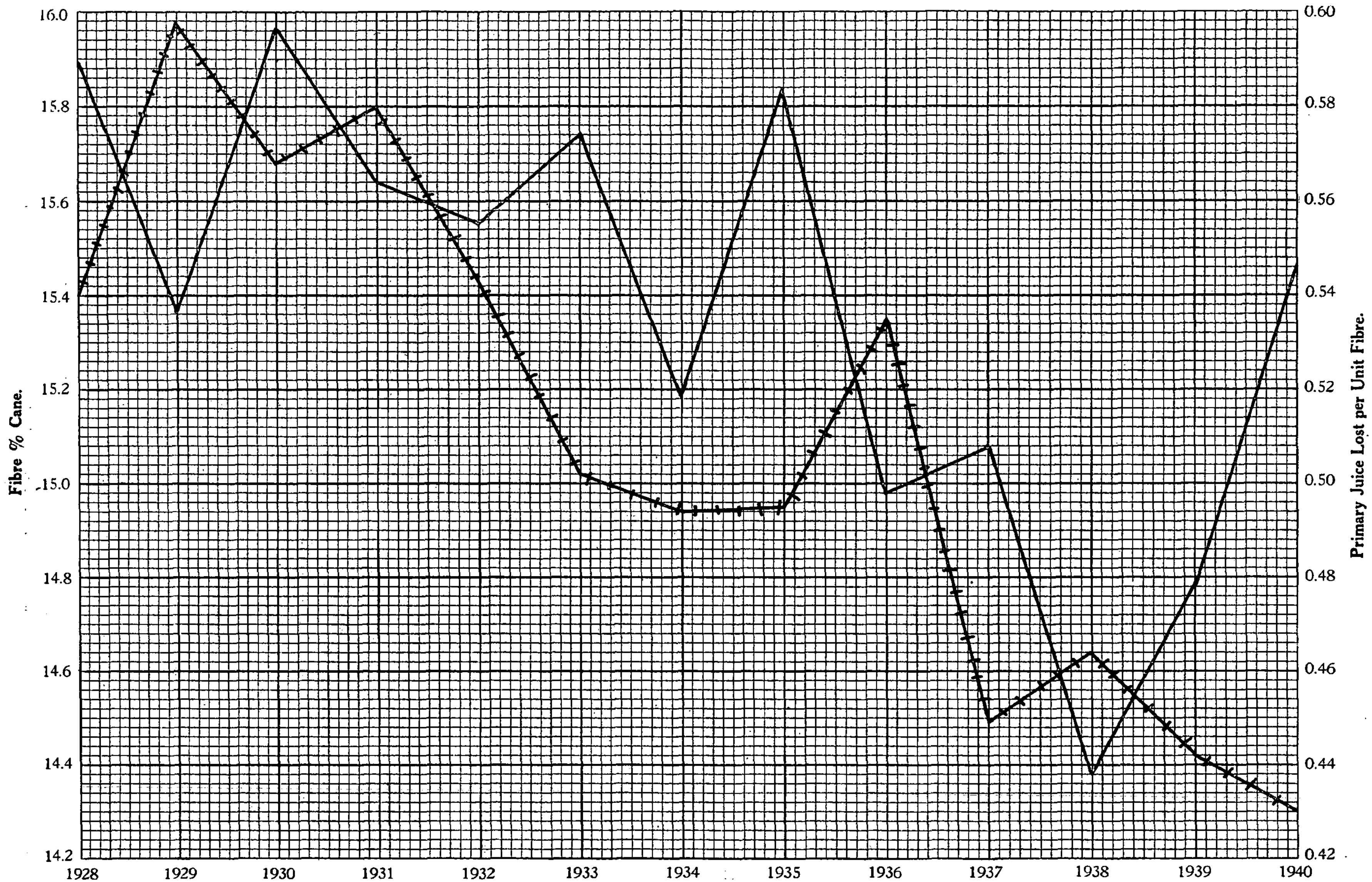
Natal Sugar Mill Results.

An examination of these results was done using the annual records of thirteen of the largest and most representative of the Natal sugar factories, all those from which the most complete information was available during the longest possible time.

This information covers the past thirteen years, the results for 1940 being available (at the time) only up to the end of November.

The same factories were always represented. Tables I and II show the annual weighted average of the results of these thirteen factories.

CHART 1.



Fibre % Cane ————————————

Primary Juice Lost per Unit Fibre —|—|—|—

CHART 2.

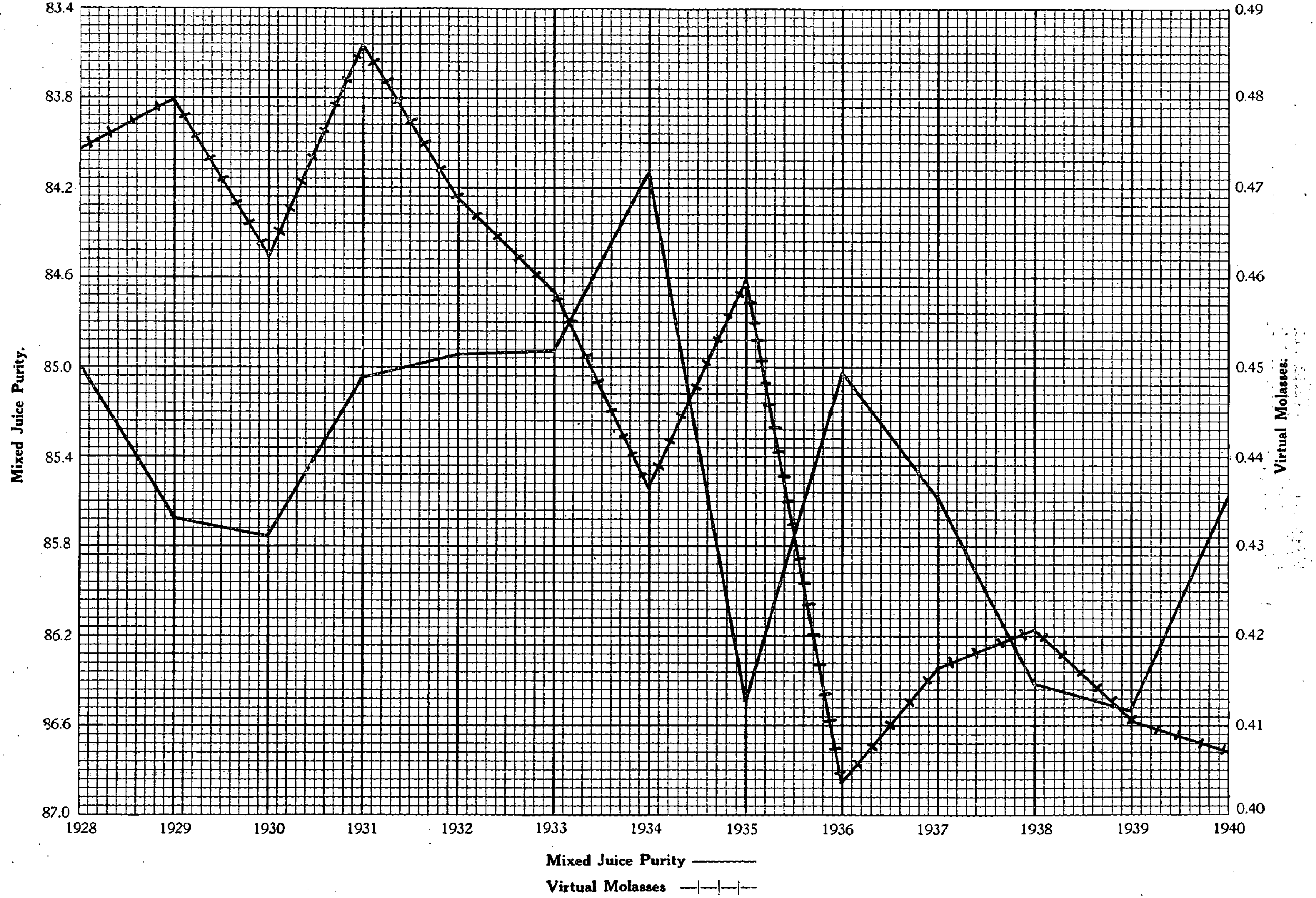

83.4
83.8
84.2
84.6
85.0
85.4
85.8
86.2
86.6
87.0
Mixed Juice Purity.
0.49
0.48
0.47
0.46
0.45
0.44
0.43
0.42
0.41
0.40
Virtual Molasses.
1928
1929
1930
1931
1932
1933
1934
1935
1936
1937
1938
1939
1940
Mixed Juice Purity
Virtual Molasses

TABLE I.

Year.	Fibre per cent. cane.	Extraction.	Primary juice lost per unit of fibre.
1928	15.887	89.788	0.5407
1929	15.358	89.144	0.5983
1930	15.963	89.220	0.5676
1931	15.636	89.258	0.5795
1932	15.554	89.999	0.5430
1933	15.737	90.632	0.5016
1934	15.181	91.165	0.4936
1935	15.840	90.693	0.4946
1936	14.978	90.587	0.5346
1937	15.085	92.022	0.4491
1938	14.382	92.808	0.4640
1939	14.787	92.322	0.4425
1940	15.466	92.125	0.4304

The second and fourth columns have been charted, Chart I.

TABLE II.

Year.	Purity mixed juice.	B.H. Recovery.	Virtual molasses.
1928	85.001	84.095	0.4741
1929	85.672	84.563	0.4800
1930	85.759	85.725	0.4623
1931	85.059	83.382	0.4861
1932	84.948	84.351	0.4690
1933	84.934	84.895	0.4584
1934	84.142	85.382	0.4368
1935	86.493	86.722	0.4595
1936	85.031	88.071	0.4038
1937	85.582	87.978	0.4164
1938	86.408	88.586	0.4207
1939	86.543	89.155	0.4109
1940	85.582	88.438	0.4070

The second and fourth columns have been charted, Chart II.

These two tables and their corresponding charts indicate immediately :—

(*a*) That there was always an element of improvement present (decrease unit loss) throughout the period.

(*b*) That in the majority of cases an increase in the fibre per cent. was associated with a decrease unit loss, and that an increase in the purity of the mixed juice was generally associated with an increase in the virtual molasses coefficient.

The element of improvement or trend can best be analysed by means of a shifting average.

TABLE III.

Average of five years.	Juice lost per unit of fibre.	Period yearly decrease.
1928/1932	0.5658	—
1929/1933	0.5580	0.0078
1930/1934	0.5371	0.0209
1931/1935	0.5224	0.0147
1932/1936	0.5135	0.0089
1933/1937	0.4947	0.0188
1934/1938	0.4872	0.0075
1935/1939	0.4769	0.0103
1936/1940	0.4641	0.0128
Average yearly decrease		0.0127

TABLE IV.

Average of five years.	Virtual molasses.	Period yearly decrease.
1928/1932	0.4743	—
1929/1933	0.4712	0.0031
1930/1934	0.4625	0.0087
1931/1935	0.4620	0.0005
1932/1936	0.4455	0.0165
1933/1937	0.4350	0.0105
1934/1938	0.4274	0.0076
1935/1939	0.4223	0.0051
1936/1940	0.4118	0.0105
Average yearly decrease		0.0078

That fibre did affect unit loss and that purity did affect virtual molasses is apparent from the tables and chart by inspection.

It does not appear logical that thirteen mills on the average would react in this manner, unless the changes in the unit loss were a natural corollary of the changes in fibre and purity.

It is therefore obvious that, before any attempt can be made to establish efficiency levels by means of unit loss or reduced efficiency, we will need to have a definite answer to the following query :—

Does a change in fibre per cent. or in the degree of purity affect *per se* the corresponding coefficient of unit loss? If so, in what manner and to what extent ?

Thus:	In 1929	over 1928	there was	a decrease	in fibre and	increase	loss.
	„ 1930	„ 1929	„	an increase	„	decrease	„
	„ 1931	„ 1930	„	a decrease	„	increase	„
	„ 1936	„ 1935	„	a decrease	„	increase	„
	„ 1938	„ 1937	„	a decrease	„	increase	„
	„ 1939	„ 1938	„	an increase	„	decrease	„
	„ 1940	„ 1939	„	an incease	„	decrease	„

Similarly:	In 1929	over 1928	there was	an increase	in purity and	increase	loss.
	„ 1934	„ 1933	„	a decrease	„	decrease	„
	„ 1935	„ 1934	„	an increase	„	increase	„
	„ 1936	„ 1935	„	a decrease	„	decrease	„
	„ 1937	„ 1936	„	an increase	„	increase	„
	„ 1938	„ 1937	„	an increase	„	increase	„
	„ 1940	„ 1939	„	a decrease	„	decrease	„

We must look for a sound method to investigate these queries.

Choice of Method: Statistical analysis.

Tables I and II consist of sets of time series of presumedly associated variables of thirteen items.

We also know that there is a time trend, a simple correlation of the associated variables will not answer directly our problem. It will be necessary to practise time trend elimination.

It was found that in practice four powers of time had to be eliminated, leaving us with only seven pairs of dependent variables, which is too short a series to be of any practical value.

We must therefore look for other methods.

Main method used.

The main method used was one developed by Dr. Stein, professor of mathematics at Natal University College, in collaboration with the writer, and fully investigated by both in its practical application on this and related problems.

We have at our disposal the yearly average fibre per cent. and its corresponding coefficient of unit loss, and the yearly average purity of mixed juice and corresponding virtual molasses coefficient for each mill. These figures are quite free from any error of estimation of process stock, and may also be said to have been determined with a great deal of accuracy. They are also free from time effect, being comparable to thirteen sets of simultaneous observations at thirteen centres.

Changes in the coefficients of unit loss may be taken to be due to—

(1) Intrinsic efficiency of a particular mill.

(2) Changes in fibre per cent. or of purity of mixed juice.

(3) Accident.

(3) may be neglected, as a measure of this factor will be afforded by the degree of significance of the correlation coefficients between the variates.

Intrinsic efficiency is eliminated, the net number of variables taken in pairs being diminished by this effect at thirteen centres and by the number of variates, that is to say two, leaving us still with over 150 pairs of associated variables. If the coefficient of co-variation between these is significant at sufficient odds (say 100 to 1) we may apply the regression equation and obtain :—

(i) The mean contribution of each mill towards the mean effect.

(ii) The mean effect and its significant error.

The results of this analysis are summarized in Tables V and VII. It is to be noted here that :—

(1) All mills contributed to the fibre effect on unit loss.

(2) That the mean fibre effect was significant at considerable odds.

(3) That all but one mill contributed to the purity effect on virtual molasses.

(4) That the mean purity effect was significant at considerable odds.

Second and Confirmatory Analysis.

The above was significant to such a high degree that the matter might well have been left there, but it was deemed prudent to confirm these results if possible. A yearly variation and a variation of extremes would also be useful.

The second analysis through the years was made. We take here the variations in the coefficients of unit loss to be due to—

(1) General improvements in plant and technique.

(2) Changes in fibre per cent. or in the purity of mixed juice.

(3) Accident.

As in the first analysis, the third factor may be discounted.

General improvements in plant and technique represented by the yearly mean are eliminated. This leaves, as in the previous case, over 150 pairs of associated variables, in each case.

If the coefficient of co-variation between these is significant at sufficient odds (say 100 to 1), we may apply the regression equation and obtain :—

(i) The mean yearly contribution of all mills towards the mean effect.

(ii) A new value of mean effect and its significant error.

The results obtained by this second analysis have been summarized in Tables VI and VIII.

The new mean value is the result of a totally different set of calculations and will either reinforce the first values obtained or tend to destroy them.

To reinforce them they will have to show :—

(i) Correlation coefficients sufficiently near to one another to indicate identity of population.

(ii) Mean values of the same order.

An inspection of the tables will show that the two analyses did in fact reinforce each other.

Tables V and VI yield a cross analysis of the effect of a change of 1 per cent. fibre on the coefficient of unit loss.

Tables VII and VIII yield a cross anlaysis of the effect of 1° purity change on the virtual molasses coefficient.

Further information.

We may now attempt to find variations in regression values for extremes of the variates.

Working with the five lowest and five highest fibre and purity years, we obtain the following set of values—the mean value already obtained being placed in the centre :—

Fibre level.	Regression value.	Correlation coefficient.	Number of pairs.
14.93	−0.035421	−0.3012	58
15.45	−0.043823	−0.3908	155
15.89	−0.045224	−0.4536	56

Purity level.	Regression value.	Correlation coefficient.	Number of pairs.
85.00	0.008584	0.3936	58
85.66	0.011623	0.4326	155
86.35	0.015592	0.4376	56

All these values except the first are significant at odds greater than 100 to 1, the first is significant at odds greater than 20 to 1.

They are therefore all valid at their respective levels. A reasonable interpretation of the results would be :—

A decrease in fibre of 1 per cent. below 14.93 increases unit loss by 0.035421.

A variation in fibre of 1 per cent. around 15.45 per cent. changes unit loss in the opposite direction by 0.043823.

An increase in fibre of 1 per cent. over 15.89 per cent. decreases the unit loss by 0.045224.

1° purity decrease below 85.00 decreases the virtual molasses coefficient by 0.008584.

1° purity variation around 85.66 changes unit loss in the same direction by 0.011623.

1° purity increase over 86.35 increases virtual molasses coefficient by 0.015592.

Application of results.

We now have :—

(i) The proof that changes in the variates fibre and purity cause a change in the corresponding coefficient of unit loss.

(ii) We know the direction of the change.

(iii) We have the magnitude of the change within a limited range.

By means of the Noël Deerr formulæ, or by quantity measurement, we find the average values of V and Mv (primary juice lost and virtual molasses) for average fibre and purity values of "F" and "J" respectively.

V_1 and M_1 for different values of fibre and purity F_1 and J_1 is calculated. We write—

$$V_1 = V - A_1$$
$$M_1 = Mv - D_1.$$

If we also take

$$x_1 = (F - F_1)$$
$$y_1 = (J - J_1),$$

we may under any circumstances write

$$A_1 = a + bx_1 + cx_1^2 + dx_1^3 + \dots$$
$$D = p + qy_1 + ry_1^2 + sy_1^3 + \dots$$

For should the coefficients of the powers x and y in the expansion of A and D prove to be all significantly equal to zero, then A and D will also be zero and the Noël Deerr formulæ for the calculation of extraction and recovery would apply without any modification.

We have seen that, in Natal at any rate, this is not the case.

By substituting Natal industrial results in the above formulæ, we find that the second, third and fourth coefficients of the powers of x and y cannot be neglected.

We have the following values :—

Coefficients of x.	Coefficients of y.
a = 0.0....	p = 0.0....
b = −0.04522	q = −0.01168
c = −0.00163	r = 0.00117
d = 0.00033	s = 0.00006
e = 0.0....	t = 0.0....

We may now write the formula for an extraction obtainable from a fibre f_1 differing from the mean:

(i) $$e_1 = \frac{[(1 - f_1)/f_1] - V_1}{(1 - f_1)/f_1}.$$

And for a recovery obtainable from a purity of mixed juice j_1 differing from the average :

$$e_1 = \frac{(j_1 - m_1)}{j_1 (1 - m_1)},$$

where $$V_1 = V - A_1,$$
$$M_1 = M_V - D_1.$$

And where

$$A_1 = -0.04552\,(F - F_1) - 0.00163\,(F - F_1^2) + 0.00033\,(F - F_1)^3.$$

$$D_1 = -0.001168\,(J - J_1) + 0.00117\,(J - J_1^2) + 0.00006\,(J - J_1)^3.$$

Summary.

The application of the Noël Deerr formulæ to Natal sugar factory conditions has been investigated, and it was found that :—

(i) A decrease in fibre per cent. resulted in an increase in the primary juice lost per unit of fibre.

(ii) An increase in the purity of the mixed juice resulted in an increase in the virtual molasses coefficient.

(iii) The extent of these changes and their variations have been determined on the results of the work of thirteen of the most representative factories in Natal over the past thirteen years.

(iv) These changes and variations are all statistically significant at considerable odds. The results of two distinct analyses confirm and reinforce each other.

(v) Noël Deerr formulæ have been modified so as to conform to Natal factory conditions.

TABLE V.

Mill No.	Mean fibre per cent.	Mean unit loss.	Mean change in unit loss due to 1 per cent. fibre increase.
1 ...	15.333	0.34525	−0.037171
4 ...	14.926	0.57362	−0.061071
5 ...	15.193	0.47946	−0.073767
6 ...	14.915	0.45588	−0.028633
8 ...	15.499	0.56273	−0.027726
10 ...	14.992	0.51911	−0.058429
11 ...	15.612	0.60635	−0.049541
12 ...	15.506	0.56978	−0.080228
14 ...	15.529	0.52728	−0.018729
15 ...	16.243	0.51491	−0.039315
16 ...	15.468	0.59995	−0.082087
18 ...	15.801	0.52580	−0.033899
21 ...	15.794	0.55449	−0.029139
Mean	15.447	0.52574	−0.04767

Correlation coefficient (partial) for over 150 pairs of variables was −0.4018. Regression is therefore significant at considerable (more than 100 to 1) odds.

Weighted average variation due to 1 per cent. increase in fibre

$$= -0.045802 \pm 0.008523.$$

TABLE VI.

Year.	Mean fibre per cent.	Mean unit loss.	Mean change in unit loss due to 1 per cent. fibre increase.
1928 ...	15.979	0.55848	−0.050299
1929 ...	15.593	0.61424	−0.070185
1930 ...	15.815	0.59509	−0.033992
1931 ...	15.795	0.58095	−0.024703
1932 ...	15.678	0.54783	−0.047746
1933 ...	15.864	0.52885	−0.009468
1934 ...	15.204	0.51758	−0.039723
1935 ...	16.008	0.50877	−0.081044
1936 ...	15.015	0.52288	−0.068748
1937 ...	15.088	0.49705	−0.103050
1938 ...	14.452	0.48727	+0.017411
1939 ...	14.881	0.45392	−0.019863
1940 ...	15.460	0.44162	−0.037290
Mean	15.449	0.52727	−0.043823

Correlation coefficient (partial) for over 150 pairs of variables was −0.3908. Regression is therefore significant at considerable (over 100 to 1) odds.

Weighted average variation due to 1 per cent. increase in fibre

$$= -0.043971 \pm 0.008456.$$

TABLE VII.

Mill No.	Mean purity of mixed juice.	Mean value of virtual molasses.	Mean change in virtual molasses due to 1° purity increase.
1 ...	85.162	0.41751	0.011495
4 ...	86.323	0.48062	0.024219
5 ...	85.623	0.41798	0.004204
6 ...	85.042	0.43688	0.011982
8 ...	85.390	0.45122	−0.004853
10 ...	84.739	0.43153	0.003589
11 ...	84.650	0.47872	0.003498
12 ...	85.757	0.43475	0.008603
14 ...	85.643	0.44088	0.014924
15 ...	86.364	0.53731	0.015729
16 ...	86.288	0.41747	0.030678
18 ...	86.696	0.47458	0.026187
21 ...	86.168	0.49421	0.000194
Mean	85.680	0.45490	0.011573

Correlation coefficient (partial) for over 150 pairs of variables was 0.4275. Regression is therefore significant at considerable (more than 100 to 1) odds.

Weighted average variation due to 1° increase in purity

$$= 0.011645 \pm 0.002011.$$

TABLE VIII.

Year.	Mean purity of mixed juice.	Mean value of virtual molasses.	Mean change in virtual molasses due to 1° purity increase.
1928 ...	85.239	0.48148	0.011902
1929 ...	86.057	0.49363	0.019370
1930 ...	85.976	0.49770	0.009284
1931 ...	85.235	0.48472	0.004897
1932 ...	85.225	0.47812	0.012487
1933 ...	85.153	0.46985	0.010757
1934 ...	84.115	0.44132	0.004511
1935 ...	86.571	0.46690	0.020939
1936 ...	85.563	0.42515	0.030091
1937 ...	85.742	0.42028	0.012175
1938 ...	86.533	0.42429	0.010990
1939 ...	86.619	0.41278	0.020790
1940 ...	85.521	0.40931	0.007960
Mean	85.658	0.45427	0.013550

Correlation coefficient (partial) for over 150 pairs of variables was 0.4326. Regression is therefore significant at considerable (more than 100 to 1) odds.

Weighted average variation due to 1° increase in purity

$$= 0.011623 \pm 0.001978.$$

Reference.

[1] Deerr, Noël (1933) : "The Reduction of Sugar Factory Results to a Common Basis of Comparison." I.S.J. **35, 214.**

The PRESIDENT said that it was an unusual paper and it would have been interesting had the author given some reasons that lead up to it.

Mr. MOBERLY congratulated Mr. Bechard on the manner in which he had handled this interesting subject. It was only a scientific statistical analysis that could unravel the truth out of a lot of apparently contradictory data. He was sorry, however, that the rather confusing Noël Deerr notations had been adhered to.

The author had pointed out that in employing the Noël Deerr formulæ to determine what portion of a difference in extraction was due to difference in fibre content, different results were obtained according to what standard fibre content was adopted. This difficulty did not invalidate the quantitative use of the formulæ for comparing extractions from different factories, because a number of extractions treated in this way would always rank in the same order of reduced extraction whatever standard fibre content was adopted, though the differences would vary.

Instead of using any arbitrary standard fibre content when comparing two extractions, in the following example the extraction obtained from one actual fibre content was reduced to another :—

Factory.	Actual fibre %	Actual extraction	V.	Reduced extraction at $f_1=10.0$	Reduced extraction at $f_1=20.0$	Difference
A	10.00	95.00	0.45	95.00	88.75	6.25
B	20.00	85.00	0.60	93.33	85.00	8.33

Here again different results were obtained, with a difference of 10.0 between the two extractions, either 6.25 or 8.33 was due to the fibre difference, and the balance due to extraction efficiency.

When mathematical equations gave ridiculous answers the reason was usually that they were asked to perform an impossible task. In this case an attempt was made to analyse the difference in extraction into two separate parts, one due to fibre difference and one due to efficiency difference. When an increment in any value was due to the simultaneous action of two causes which operate not only on the original value, but also upon each other, it was not mathematically possible to separate the effects of the two causes.

A simple analogy would be the case where two vehicles used different quantities of fuel, and the fuels were different in price. The cost of fuel per mile therefore differed and at first glance it appeared simple to find how much of this difference was due to the difference in fuel cost. Mr. Moberly gave the following example :—

Vehicle.	m.p.g.	Fuel price per gal.	Fuel cost per mile.	Fuel at 2/6	Fuel at 3/4	Difference
A	30	2/6	1d.	1d.	$1\frac{1}{3}$d.	$\frac{1}{3}$d.
B	20	3/4	2d.	$1\frac{1}{2}$d.	2d.	$\frac{1}{2}$d.
			Difference	$\frac{1}{2}$d.	$\frac{2}{3}$d.	

The difference of 1d. per mile might therefore be divided into $\frac{1}{2}$d. for mileage and $\frac{1}{2}$d. for fuel cost, or into $\frac{2}{3}$d. for mileage and $\frac{1}{3}$d. for fuel cost. It would not, however, be fair to say that the determination of cost of fuel per gallon was valueless in considering running costs. Neither would it be just to say that the Noël Deerr formulæ were valueless in considering the causes of extraction difference.

Mr. DODDS said that he did not propose to discuss the theoretical foundation of the Noël Deerr formulæ or Mr. Bechard's suggested modifications, since he was not sufficiently qualified to do so. He considered the Noël Deerr formulæ a convenient and useful method of comparison of factory efficiencies brought down to a uniform basis, and they had been used for several years in the Annual Summary. The standards which were used, 12.5 per cent. fibre content and 85° purity, were reasonably close to the world averages. In applying these formulæ the assumption was made that the nature of the fibre and of the non-sugars in solution, whether low or high, were the same, and that was not altogether justified. It would be interesting to apply the Deerr formulæ and compare the results with that obtained by using Mr. Bechard's modification. In Hawaii they considered milling loss as a more accurate reflection of milling performance than extraction.

Mr. DU TOIT asked how it was possible to obtain over 150 pairs of variants from Table VI. where there were only 13 fibre figures. He took it that each was the average of thirteen factories, but even then there were only 13 individual results. He also wanted to know how the main method differed from the second and confirmatory method, and whether the same figures were not used.

Dr. McMARTIN said that this paper was rather outside his sphere of activity. He could not see either how 150 pairs of variables were obtained. He wanted to know what figures were used to obtain the correlation coefficient and where the tests of significance were.

Mr. BECHARD said there were 13 mills reporting for 13 years and therefore there were 169 sets of simultaneous observations when taken in pairs.

In the first analysis each mill was weighed up against the other 12 yearly. There were therefore for each year 13 sets of variables in each table.

This was done 13 times for each year. Time trend was absent as each year was taken by itself. The finite sum of the variations from the mean was taken over 169 pairs of associated variables in each case, giving a gross correlation over 169 items. The mean efficiency of each mill was eliminated for 13 centres. Two more had to be deducted from 169, leaving 154 pairs of associated variables (over 150). Tables V. and VII. therefore represented the mean contribution of each mill through 13 years; each line being the mean of 13 results.

The second analysis, although from the same data, was quite different. There was for each pair of variables a time series of 13 items at each mill. There were 13 time series of 13 items for each pair of associated variables—169 pairs of associated variables in each case. Each mill was analysed separately by itself. The finite sum of the variations from the means was derived from 13 time series at 13 centres (169 items), giving a gross correlation over these 169 pairs of variables in each case. The mean yearly improvement was eliminated; that meant 13 yearly effects plus the two variables under examination leaving again 154 net pairs of variables. The calculation was different, as in this case the result of each mill through 13 years was analysed and the elimination was practised on the yearly improvement.

Tables VI. and VIII. represented the mean effect of 13 mills during each year. Each line was the mean of 13 results.

It was impossible to print all the calculations.

Dr. STEIN explained that there was no time factor in these calculations when the data of the same year were used. In working the reduced extraction and recovery the order was usually kept irrespective of what standard was taken, but the figures so obtained were meaningless and there was no object in giving them.

Mr. RAULT said that he was rather at sea with the new phraseology used in this paper. His experience was not that a certain part of fibre carried away a certain definite amount of sugar. He was not inclined to attach undue importance to the fibre content of cane as invariably a measure of milling value. Apparently other variables in the composition of the cane which were not measurable at the present stage had a disturbing effect on the value of the mathematical formulæ which took unit fibre as a basis and neglected all other things. In 1940 they obtained the best extraction and the highest crushing rate with a fibre of over 16 per cent. The worst extraction was obtained at the end of the season when less cane was crushed.

Dr. STEIN said that he had been very much impressed by Mr. Bechard's paper when he had seen it first. Mr. Bechard asked the question whether all fibres carried away the same quantity of juice per unit fibre irrespective of the amount of fibre, and he found that that was not the case. The paper showed that the more the fibre the less the quantity of loss per unit fibre. The speaker had not checked the paper but he thought it gave a lot of useful information.

Mr. BECHARD thanked those who had participated in the discussion and more especially Dr. Stein. He did not see the use of reduced extraction and reduced recovery. In Java they were using the absolute juice lost per unit fibre and that, he thought, was all that was necessary. The virtual molasses coefficient could be applied in the same way.

Mr. DYMOND commented on the paper in writing as follows :—

"Mr. Bechard's paper supplies a long-felt want in proving statistically what the factory technologists have noted ever since the introduction of non-Uba canes into this country.

That a unit of fibre does not take away the same amount of primary juice in practical milling is due, in my opinion, to the difficulties of a mixed cane supply engendering, as it does, the crushing and sampling of large and small individual consignments of high and low fibred cane separately, but with a predominance of high fibre to which the mills must be set; and also to large variations in the "quality" of fibre in the individual varieties, which is accentuated under the peculiar climatic and soil conditions prevalent in Natal.

"These are the causes; Mr. Bechard has proved their effect.

"In the same way, but not to the same degree, a unit of impurity does not cause the same loss of sucrose. Here the causes are more difficult to ascertain with accuracy. It is known that phosphate deficiency in soils and in cane juices leads to difficulties in clarification. In Natal this condition exists independently of the variety crushed. As a consequence, the amount of chemicals used in clarification exceeds that in any part of the world, and this is quite independent of variety. It is further known that the higher ranges of purity result in an average flattening curve of recovery. This, in my opinion, is largely caused by inadequate boiling house capacities to deal with the peculiarities of juice and sugar manufacture in Natal. By saying inadequate I do not necessarily mean economic, for there is a limit to the economic recovery of theoretical values.

"I think it important that Mr. Bechard's paper be subjected to the closest checking by competent authorities, so that those who believe that the "label" on a cane stalk is the primary element in sugar manufacture will be forced to believe that there are more things in sugar manufacture in Natal than they knew of.

"I would like to congratulate Mr. Bechard on his paper."

The PRESIDENT thanked Mr. Bechard for his interesting paper, and Dr. Stein for his presence and clear explanations. He suggested that this paper be sent to Noël Deerr to obtain his comments. He asked the Conference to record a hearty vote of thanks to Mr. Bechard for the work he has done to prepare his paper.

(*Applause*).