ON EFFICIENCY INDICES IN SUGAR MILLING

By Prof. P. STEIN.

INTRODUCTION.

The indices commonly used to estimate the efficiency of the crushing process in the sugar industry are (1) the fraction of the juices in the cane which is extracted by crushing, and (2) the quantity of juices per unit quantity of fibre left in the fibre. Calling these e and v, 100e is the percentage extraction, while v is one of the Noël Deerr coefficients.

If f is the fibre in unit quantity of cane, e, v and f are connected by the formula[2]:—

$$v = \frac{(1-e)(1-f)}{f}.$$

Generally speaking, efficiency is high if e is high and v is low. However, in order to correctly appreciate in what way a high value of e or a low value of v measure efficiency, it is necessary to know how e and v vary as f varies. An interesting and original paper by R. M. Bechard[1] gives valuable information on this topic. Analyzing statistical data in the South African sugar industry, Mr. Bechard concludes that v increases as f decreases and obtains a formula connecting v and f. One of the objects of this paper is to reconsider Mr. Bechard's statistical data and the formula he derives.

It appeared to the writer that simpler results connecting e, v and f might be obtained by attempting to get the statistical relation between e and f rather than between v and f. The hypothesis was made that e was a constant, i.e., independent of f. This hypothesis was tested using Mr. Bechard's calculations. It was found that although his final formula is not in complete agreement with the hypothesis, his preliminary results are.

Reasons are given why greater weight is to be placed on the preliminary results than on the final formula, and it is concluded that the hypothesis that e is independent of f is well supported by Mr. Bechard's statistics. Taking e as a constant, the variation of v with f can then be calculated by the Noël Deerr formula.

The calculations and arguments are given in Part 1 of this paper. Part 2 consists of some suggestions as to how the indices e and v may legitimately be used to measure efficiency. These suggestions are based on the theory that e is independent of fibre content.

Part 3 is rather theoretical. In it the writer attempts to initiate a mathematical theory of costing in sugar recovery.

If £a is the cost of extracting juices per ton juices extracted, then a is a useful efficiency index. However, in comparing one mill with another the use of this index may lead to misleading information, for the reason that the cost per ton depends not only on efficiency of staff and plant, but also on the percentage extraction. There is the law of diminishing returns, and a mill with a higher percentage extraction may be expected to have a higher cost per ton of juices extracted.

Making some plausible hypotheses, an index of costing is obtained which purports to be a basic index, i.e., one in which the factor of extraction percentage is eliminated. This index, or some such index which may subsequently be obtained by further study of the process of juice extraction, is a more satisfactory index of the efficiency of staff and plant than the figure giving the cost per ton of juices extracted.

Owing to the law of diminishing returns, a stage may be reached in the development of a mill when further expenditure to increase percentage extraction is economically unjustifiable. A figure is obtained which purports to give the maximum percentage extraction economically attainable in any given mill, working a definite average quality cane. A figure is next obtained of the percentage of juices lost which may be economically recovered.

My thanks are due to Mr. R. M. Bechard for the kindly manner in which he assisted me in many ways in the preparation of this paper.

PART 1.

In his paper Mr. Bechard[1] investigates the effect of fibre content on the value of the Noël Deerr coefficient v. He finds that in general v increases as the fibre content diminishes, and calculates the average regression of v for 13 mills over a period of thirteen years. This average regression value works out at −0.043971 for every one per cent. of increase in fibre. He also obtains a formula for the variation of v. On referring to his manuscript, Mr. Bechard gave me a result which, with a slight change in notation, can be stated as follows :—

If f is the fibre content of a cane and f is near 0.1545, and f_1 the fibre content of another cane, then if $x = 100(f_1 - f)$, v and v_1 the corresponding Noël Deerr coefficients, then

$$v_1 - v = -0.042788x - 0.002599x^2 - 0.000519x^3 \ldots \quad (1)$$

Suppose we make the hypothesis that e is constant and attempt to check this hypothesis with the help of Mr. Bechard's figures. We have—

$$v = \frac{(1-e)(1-f)}{f},\quad v_1 = \frac{(1-e)(1-f_1)}{f_1},\quad x = 100(f_1 - f),$$

we have

$$\frac{v_1}{v} = \frac{1-f_1}{f_1} \times \frac{f}{1-f} = \frac{1-f-\frac{x}{100}}{f+\frac{x}{100}} \times \frac{f}{1-f} = \frac{1-\frac{x}{100(1-f)}}{1+\frac{x}{100f}}.$$

This gives

$$v_1 = v\left\{1 - \frac{x}{100} \times \frac{1}{f(1-f)} + \frac{x^2}{100^2} \times \frac{1}{f^2(1-f)} - \frac{x^3}{100^3} \times \frac{1}{f^3(1-f)} \ldots\right\}$$

If f = 0.1545 this gives

$$v_1 - v = v(-0.07654x + 0.00490x^2 - 0.00033x^3).$$

The average of v for the 13 mills over the period of 13 years is given by Mr. Bechard as 0.52727. Taking this as approximately equal to v corresponding to the average fibre 0.1545, we get :—

$$v_1 - v = -0.04035x + 0.00261x^2 - 0.00017x^3. \ldots \quad (2)$$

If we compare this with Mr. Bechard's formula (1), neglecting the x^3 term, we find the coefficients of x−0.04035 and −0.04279 agree sufficiently. However, the coefficients of x^2, although almost equal in value, are different in sign. This discrepancy is too serious for Mr. Bechard's formula to be considered, as well supporting the theory that e is constant.

The hypothesis that the same process applied to a cane will extract the same percentage of the juices in the cane is, however, very attractive on theoretical grounds. It is thus worth while to scrutinize Mr. Bechard's calculations more closely.

Mr. Bechard based his formula on the statistics obtained from 13 mills over a period of 13 years. His main problem was to eliminate the time trend. His method was as follows. He calculated the average regression coefficients over the 13 years, over the 5 years with the lowest fibre content, over the 5 years with the highest fibre content, and used a method of interpolation.

He had the table :—

Fibre level.	Regression value.
0.1493	− 0.035421
0.1545	− 0.043971
0.1589	− 0.045224

His method of eliminating the time trend in obtaining the regression values seems to be entirely satisfactory and one can accept the regression values obtained.

If we now write

$$v_1 - v = bx + cx^2 + dx^3, \text{ where } x = 100\,(f_1 - f)$$

and attempt to find b, c, d (as Mr. Bechard did) by interpolation from the above table, as we cannot be sure that b, c, d are not affected by the time trend, we must be sure that our regression coefficients are independent of the time trend.

Unfortunately the five years of lowest fibre all occur in the last seven of the 13 years used; also the five years of highest fibre all occur in the first eight of these years. Hence, one may conclude that although the regression values are themselves satisfactory, they cannot be used to find b, c, d without the risk of error. We thus accept Mr. Bechard's table of regression values, but consider his formula not sufficiently accurate.

Returning to the theory of constant extraction, bearing in mind the smallness of the coefficient of x^3, we may take the average regression as given by this formula at the fibre level 0.1545 and $v = 0.52727$ as the coefficient of x in equation (2), i.e., 0.04035.

The average value of v for the five years of lowest fibre content is 0.4957 and the fibre level is 0.1493. Carrying out the calculations by which equation (2) was obtained, the coefficient of x (and so the average regression coefficient) works out as -0.03902. For the five years of highest fibre level $f = 0.1589$, $v = 0.5544$, and the regression coefficient works out as -0.04143.

We have the following table:—

REGRESSION VALUES.

Fibre	(1)	(2)	(3)	(1)−(2)	(3)−(2)
0.1493	−0.03542,	−0.03902,	−0.03990,	+0.00360,	−0.00088
0.1545	−0.04397,	−0.04035,	−0.04375,	−0.00362,	−0.00340
0.1589	−0.04522,	−0.04143,	−0.04299,	−0.00379,	−0.00156

Column (1) gives Mr. Bechard's weighted averages. Column (2) represents the regressions as calculated by the constant extraction hypothesis. Column (3) is an unweighted average from Mr. Bechard's figures.

It will be seen that the differences between the weighted averages and the computed figures are in no single case as high as 10 per cent. of the values of the averages. Working with unweighted averages the differences are even less. Mr. Bechard gives as the standard error of the value -0.04397 as ± 0.008456. The standard errors in other cases, obtained from fewer data, should not be less. Hence the computed values are well within the standard errors.

We contend that Mr. Bechard's statistical data is good evidence supporting the theory that e is constant.

PART 2.

Of the two indices e and v, we may assume that e is independent of fibre content, while v varies as the fibre content varies. e is a simple efficiency index. Disregarding the efficiency of plant and staff, e measures in particular the development of a mill.

v is not a mill constant. v varies as f varies according to the formula

$$v = \frac{(1-e)\,(1-f)}{f},$$

where e is a mill constant.

v diminishes as f increases. The average value of v may, however, be used to give useful information. In the process of extracting juices from the cane, as the quantity of juices accompanying unit fibre gets less and less, more and more effort (and so cost) is required to extract a given quantity of juices from the fibre.

Suppose we have two mills, which have the same value of e (and so may be taken to be equally highly developed), and the one mill deals with a cane of a lower average fibre content, it then follows that this mill has a higher value of v. In consequence it would cost less to extract additional juices in this mill than in the other. Hence, it would be more profitable to develop this mill further to extract additional juices.

v may be said to give a measure of the efficiency of a mill relative to the average fibre content of the cane it handles. Thus two mills with the same average value of v are equally efficient, bearing in mind the average value of the fibre content of the cane they handle, even if the values of e for these mills are different.

Useful comparisons might be made between the value of v for any one mill and the value for other mills in this and any other country.

Bearing in mind the relative values of sugar and the relative economic efficiency of labour, such a comparison would be a help in deciding whether additional expenditure in further development would be economically desirable.

By such comparison an estimate as to the minimum value of v and the maximum value of e attainable in any one mill might be obtained approximately well. A direct theoretical solution of these two problems is attempted in the next part.

PART 3.

As proved in the first part, the crushing plant of a mill works so that the percentage of the juices associated with a given quantity of fibre which is extracted from the fibre is the same for all juice content of the fibre.

We make the plausible hypothesis that a *portion* of a crushing plant acts in the same way. That is to say, that irrespective of the juice content of a fibre, if a quantity of fibre passes over a portion of the crushing plant the same percentage of the juices is extracted.

Consider now a small section of the plant. Suppose J is the juice content of the fibre. Let unit quantity of fibre pass over this portion, and let j be the quantity of juices extracted. We have $\frac{j}{J}$ as a constant. Let j_1 be the juices extracted when the juices content of the fibre is 1. Then

$$\frac{j}{J} = j_1.$$

We now suppose that the cost of running through the fibre is the same whether it contains J or 1 unit of juices. Hence the cost of obtaining j units of juices at concentration J is the same as the cost of obtaining j_1 units at concentration 1. Since $j = j_1 J$, if k is the cost of obtaining 1 unit of juices at concentration 1, then the cost of obtaining 1 unit of juices at concentration J is $\frac{k}{J}$.

We now suppose further that all portions of the crushing plant are equally efficient, that is to say that it costs the same sum of money to put a quantity of fibre through any portion for which there is the same percentage extraction. This gives that the cost of extraction of juices per unit quantity of juices at concentration J is $\frac{k}{J}$, where k is a mill constant.

If we start with unit quantity of fibre at concentration J_0 and extract juices from it till the concentration is v, then a simple mathematical calculation,

$$-\int_{J_0}^{v} \frac{k}{J}\,dJ = \int_{v}^{J_0} \frac{k}{J}\,dJ = 2.3026\,k \log \frac{J_0}{v}$$

will show that the total cost of extracting the juices is 2.3026 $k \log \frac{J_0}{v}$ where the logarithm means the common logarithm.

If e is the extraction, f the fibre content, then we have

$$J_0 = \frac{1-f}{f}, \quad v = \frac{(1-e)\,(1-f)}{f}.$$

Substituting these values, we get that the cost of extracting the juices from unit quantity of fibre is

$$2.3026\,k \log \frac{1}{1-e}.$$

The total quantity of juice extracted is $\frac{e(1-f)}{f}$ and hence the cost of extracting unit quantity of juice is

$$\frac{2.3026\ f}{e(1-f)} \times k \log \frac{1}{1-e}.$$

If now the cost of extracting unit quantity of juice is known from a costing system in the mill to be £a, we get

$$k = £ \frac{e(1-f)\ a}{2.3026\ f \log \frac{1}{1-e}}.$$

k is a basic efficiency index. It gives the cost of obtaining unit quantity of juices at unit concentration. It is an index of efficiency of a plant without regard to development.

The cost of extracting unit quantity of juices at concentration J is $\frac{k}{J}$. Let £b be the value of unit quantity of juices after its extraction. Then if v_0 be the minimum of J for economic crushing we have $b = \frac{k}{v_0}$, giving

$$v_0 = \frac{e(1-f)}{2.3026\ f \log \frac{1}{1-e}} \times \frac{a}{b}.$$

If e_0 be the maximum extraction economically attainable we calculate e_0 by the Noël Deerr equation and obtain

$$e_0 = 1 - \frac{f v_0}{1-f} = 1 - \frac{e}{2.3026 \log \frac{1}{1-e}} \times \frac{a}{b}.$$

As a useful milling index it is suggested that we calculate the value of

$$100\ (e_0 - e).$$

This gives the percentage of extraction that may be regarded as lost and which is economically recoverable. This index takes into account fibre content, costs and value of sugar, and is thus an overall index of economic development.

As a subsidiary overall index the value of $\frac{v_0}{v}$ might be taken. The formula giving this is

$$\frac{v_0}{v} = \frac{e}{2.3026\ (1-e) \log (\frac{1}{1-e})} \times \frac{a}{b}.$$

References.

[1] Bechard, R. M. (1941): "Natal Sugar Mill Results." Proc. S.A. Sugar Tech. Assoc., **15**, 52.

[2] Deerr, Noël (1933): "The Reduction of Sugar Factory Results to a Common Basis of Comparison." I.S.J., **35**, 214.

Mr. DU TOIT said that in the first part of his paper Prof. Stein obtained the remarkable result that extraction was independent of fibre. This result was derived indirectly and the speaker would have preferred a direct proof. Secondly, this relationship was proved for a very limited range of fibre (14.93 per cent. to 15.89 per cent.). Prof. Stein, however, saw no reason why extraction should not remain constant for a wider range of fibre.

In the second part of his paper Prof. Stein accepted v, primary juice loss per unit fibre, as an efficiency index which varied with fibre. Although not mentioned in his paper, Prof. Stein was of the opinion that reduced extraction was a figure both unnecessary and wrong. That, too, was the conclusion that Mr. Bechard came to in his paper "Natal Sugar Mill Results" presented to the Conference last year. The value of primary juice loss as a basis of comparison had been realised for a long time and it was always included in our Annual Summary. Mr. du Toit maintained, however, that if primary juice loss were a useful index, then reduced extraction must be one as well, as these two figures were derived in a similar way.

If two mills equally developed (having the same e, say 0.92) were taken, then if mill (1) were crushing cane of 15 per cent. fibre and mill (2) cane of 12.5 per cent. fibre, than v for mill (1) would be 0.4533 and v for mill (2) would be 0.5600. If mill (2) were, however, further developed to give an extraction of 93.524 per cent., then its efficiency would become 0.4533 and the two mills would then be equally efficient. Now 93.524 was, of course, the extraction of mill (1) reduced to 12.5 per cent. fibre. Mills having the same efficiency v must have the same reduced extraction and vice versa. These two measures were inseparably bound up by formula and definition. It did not follow, however, that the cost of attaining the same efficiency was the same.

Part three of Prof. Stein's paper dealt with something completely new, e_0, the maximum extraction economically possible. This index could be of the greatest value to the factories, but then it was absolutely necessary that it should be taken up and used. The actual test of its validity would only be found in practical application. Should its value be established in this way it would bring other developments. An exact system of costing would be necessary and it might prove the desirability of an economist as part of the factory personnel.

The following example was worked out by Mr. du Toit, who pointed out the figures of costs were purely arbitrary :—

Mill extraction 92.0 per cent. Cost of extracting 1 ton of juice = 3.7/-.

Cost of further recovering the sugar in 1 ton of juice = 5/-.

Total value of recoverable sugar in 1 ton of juice = 27/-.

Then the value of 1 ton of juice = 27/- − 5/- = 22/-.

$$e_0 = 1 - \frac{e}{2.3026 \log \frac{1}{1-e}} \times \frac{a}{b}$$

$$= 1 - \frac{0.92}{2.3026 \log \frac{1}{0.08}} \times \frac{3.7}{22}$$

$$= 1 - \frac{0.92}{2.3026 \log 12.5} \times \frac{3.7}{22}$$

$$= 1 - \frac{0.92}{2.3026 \times 1.0969} \times \frac{3.7}{22}$$

$$= 1 - 0.0613$$

$$= 0.9387.$$

Therefore $100\ (e_0 - e) = 100\ (0.9387 - 0.92) = 1.87$.

This mill, therefore, offered economic possibilities for increasing its extraction. If e_0 were equal to e, then extraction was at its maximum, while the process was uneconomical if e were greater than e_0.

The value e_0 did not only depend on the cost of extraction, but also on the cost of further recovery of the sugar, and this might be affected by increasing the extraction.

It was perhaps unfortunate the Prof. Stein had used e_0 for maximum economic extraction, as this was also the notation used by Noël Deerr for reduced overall recovery.

Mr. BECHARD said that what Prof. Stein called plausible hypotheses were not hypotheses at all but facts. It was due to these facts that mill manufacturers could guarantee that a particular unit would extract a certain percentage of the residual juice in the material received.

The cost of running a mill plant depended primarily on operation and maintenance costs, plus interest, depreciation, supervision and administrative costs. These costs were not affected to any extent by juice concentration.

Prof. Stein's paper gave the answer to such questions as whether further development was justified, which of the known forms of development would be the best, and how this would affect the balance sheet.

A milling plant of six milling units obtained an extraction of 92.82 from a cane of 15.43 per cent. fibre. The coefficient of unit loss v was therefore 0.3935. The cost of milling one ton of juice was £0.091 and the value of one ton of juice £1.06.

$$v_0 = \frac{e\,(1-f)}{2.3026\ f\ \log \frac{1}{1-e}} \times \frac{a}{b}.$$

$$= 0.166.$$

The limit of profitable extraction at equal efficiency would therefore be 96.97.

The last four units of this particular plant averaged an extraction of 25 per cent. of the residual juice. It was therefore possible to prepare the following table :—

No. of units.	Extraction by unit.	Extraction after unit.	Residue.
6	—	92.82	7.18
7	1.80	94.62	5.38
8	1.34	95.96	4.04
9	1.01	96.97	3.03

It would therefore be necessary to add another three units to attain the limit of development of the plant. The juice extracted would increase from 78.5 to 82.0 per cent. and the cost per ton of juice to

$$£0.091 \times \frac{9}{6} \times \frac{78.5}{82.0} = £0.130.$$

The value of juice would also decrease to £1.058 and the following balance sheet could be prepared :—

Juice extracted per 100 cane.	Cost per ton of juice.	Cost per 100 cane.	Return in juice value.
78.50	£0.091	7.14	83.21
82.00	£0.130	10.66	86.76
Increase	—	3.52	3.55

These formulæ could also be used to see how much cost would have to increase due to industrial legislation or increase cost of material before it would be necessary to curtail development. This could be done by taking the following values :—

v_1 unit loss = 0.3935,

a_1 limit cost,

b_1 value at cost $a_1 = b - (a_1 - a)$.

From this it followed that the maximum permissible cost was £0.195.