ACHIEVING AND CONTROLLING VACUUM IN PROCESS VESSELS USING CONDENSERS AND VACUUM PUMPS

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Abstract

Process vessels in the sugar industry which operate under vacuum, normally use a combination of a direct contact condenser and a liquid ring vacuum pump to achieve and maintain the required operating pressure. The performance of these vacuum systems depends on the complex interaction of operating conditions, control systems and equipment performance characteristics, sometimes producing unexpected behaviour. To assist in the understanding of vacuum system behaviour, simple mathematical models of the relevant pieces of equipment can be combined to simulate the effect of range of factors on overall performance. The models currently used represent ideal behaviour of a direct contact condenser and a positive displacement vacuum pump. These ideal models can be extended to model the non-ideal behaviour of real systems and options for doing this are discussed. Examples are provided of how even the relatively simple ideal models can help in understanding the behaviour of real systems.

Keywords: vacuum, vacuum pump, condenser, control

Introduction

Much of the evaporation in sugar factories takes place under vacuum both for reasons of steam economy and to reduce thermal degradation of sucrose and other organic compounds. The vacuum conditions in final effect evaporators and pans are normally achieved by a combination of a condenser and a vacuum pump. Within the South African sugar industry a common approach is to use a direct contact counter current condenser and a liquid ring vacuum pump. Since it is important to operate at a constant pressure, most importantly in pans, a control system is necessary to achieve this. Figure 1 shows a conventional arrangement of condenser and vacuum pump with a control system which regulates the cooling water flow to maintain the required operating pressure.
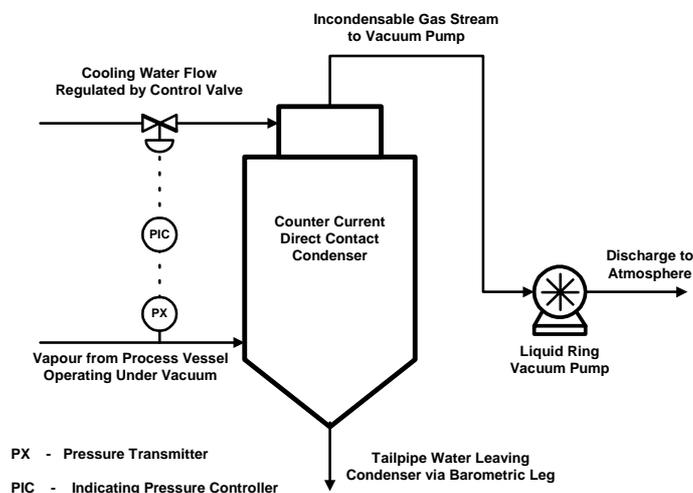

Figure 1. Arrangement of condenser, vacuum pump and control system.

The operating pressure is measured with an absolute pressure transmitter which sends a signal to the pressure controller. Expressed simply, a reduced cooling water flow will result in less condensation, an increase in load on the vacuum pump and thus a rise in absolute pressure in the vessel. The controller is thus configured to increase the injection water flow when the pressure in the vessel increases and decrease the flow when the pressure decreases.

Fundamentals – The Gas Laws

To understand the behaviour of vacuum systems it is necessary to have an understanding of the behaviour of gasses in terms of temperature, pressure, volume, mass and composition. The basic relationships are well described by the 'gas laws' of elementary physical chemistry viz:
- Charles' Law
- Boyle's Law
- Avagadro's Law
- The Ideal Gas Equation (which combines the above three laws)
- Dalton's Law of Partial Pressures

The behaviour of saturated steam (water vapour) is also an important consideration in understanding condenser behaviour as this addresses the equilibrium between water in the liquid and vapour phases.

The gas laws, the properties of saturated steam and important relationships that can be derived from these fundamentals are summarised in Appendix 1.

The models used in this paper assume that the gaseous streams consist of only two components, namely, air and water. Air is clearly not a true chemical compound but its behaviour approximates such, with a molecular mass of 29.

Model of an ideal condenser

The counter current nature of the condenser is represented in Figure 2 which shows the four streams which define the behaviour of the condenser.
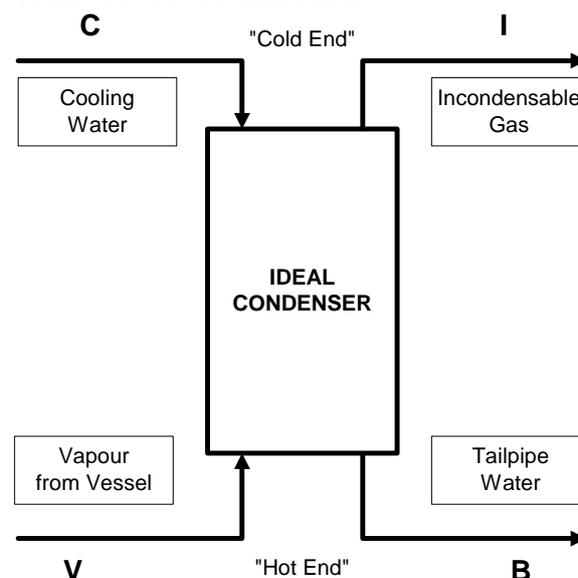

Figure 2. Counter current condenser showing inlet and outlet streams.

The ideal condenser model is based on the following assumptions:

- The cooling water contains a known portion of air (a value of 34ppm is commonly assumed) which is liberated inside the condenser due to the reduced pressure.
- The vapour from the process vessel also contains a proportion of air which is a combination of that entering with the feed into the vessel and from air leaks into the vessel.
- The total of all the air entering with both of these streams leaves via the incondensable gas stream, and there is thus no air in the tailpipe water stream.
- The condenser itself has no heat or mass, losses or gains (e.g. it has no air leaks or cooling from its surface) and thus mass and heat balances can be performed over the two entering and two leaving streams.
- The condenser operates as a perfect counter current contacting device, which means that equilibrium is achieved at one of the two 'ends'. Thus, either the incondensable gas stream leaves at the cooling water temperature, or the tailpipe water leaves at the temperature of the incoming vapour. At one critical condition when the cooling water flow is perfectly matched to the incoming vapour flow, equilibrium conditions are achieved at both 'ends' of the condenser.
- All the liquid leaves via the tailpipe (i.e. there is no carryover of water droplets in the incondensable gas stream which consists of only water vapour and air)
- There is no drop in pressure within the condenser and the incondensable gas leaves at the pressure of the incoming vapour.

For this rather simple system, it turns out that the calculations are relatively complex and thus it is important for clarity to use a simple and consistent system to name streams and their properties. The system used in this work is described in the Nomenclature section. The condenser model based on these assumptions and nomenclature is summarised in the flow diagram shown in Figure 3.
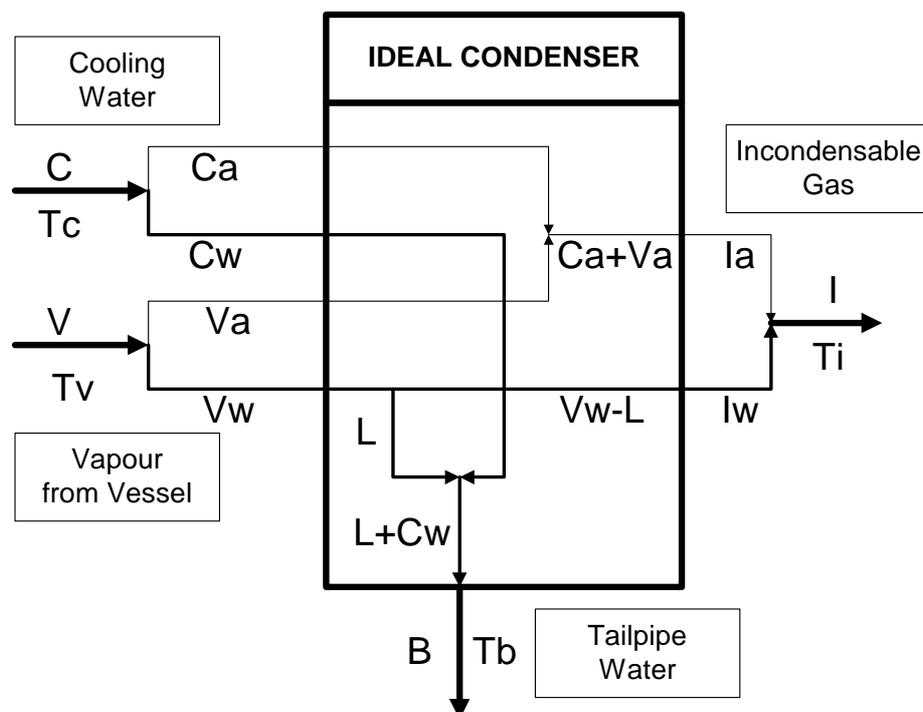

Figure 3. Flow diagram showing the basis of the model of an ideal condenser.

To understand the flow diagram in Figure 3, it is necessary to know that *C*, *V*, *I*, *B* and *L* refer to total mass flow rates of cooling water, vapour, incondensable gas, tailpipe water and condensed vapour respectively. The lower case letters *a*, and *w*, when appended, define the mass flow of air and water respectively in a particular stream. The letter *T* refers to the temperature of a particular stream, as defined by the appended lower case letter.

This flow diagram provides the guide for setting up the heat and mass balance equations which define the ideal condenser model. The calculations are structured so that the details of the vapour stream and the cooling water stream need to be fully specified. The details of the tailpipe water and incondensable gas streams must then be calculated by the equations of the model. The use of SI units throughout calculations greatly simplifies the balances and the relationships between mass and energy flows.

The formulation and solution of the equations is described in detail in Appendix 2. It turns out that the calculation procedures are different for the two possible conditions of equilibrium at either the 'hot end' or the 'cold end' of the condenser. By performing the calculations for one of the possible conditions it is possible to determine from the results of the calculation whether this is in fact a feasible operating condition and then if it is not, to perform the calculations for the other possible condition.

It also transpires that it is not possible to formulate the equations in an explicit form, but it is possible to structure the appropriate equations so that they can be solved by simple iteration until the result converges. The model equations described in Appendix 2 are relatively easy to implement on an Excel spreadsheet, with circular references used to perform the iterations necessary to converge to a solution.

Behaviour of an ideal condenser

To understand the behaviour of an ideal condenser independent of the vacuum pump it may be connected to, it is necessary to consider the incondensable gas discharging into a vacuum system which operates at constant pressure and has infinite capacity (approximated by a very large central vacuum system).

A spreadsheet based version of the ideal condenser model has been used to calculate the performance of a condenser for the following assumed conditions:

Total mass flow of vapour into the condenser	15 ton/h
Operating pressure	15 kPa absolute
Concentration of air in vapour entering the condenser	3000 ppm
Concentration of air in cooling water	35 ppm
Cooling water temperature	35ºC

By running the model for a large number of different cooling water flow rates it is possible to generate the performance graph shown in Figure 4.

The temperature results are presented as 'approach' temperatures. The tailpipe approach temperature is the difference between the tailpipe temperature and the vapour temperature. The incondensables approach temperature is the difference between the incondensable gas temperature and the cooling water temperature.
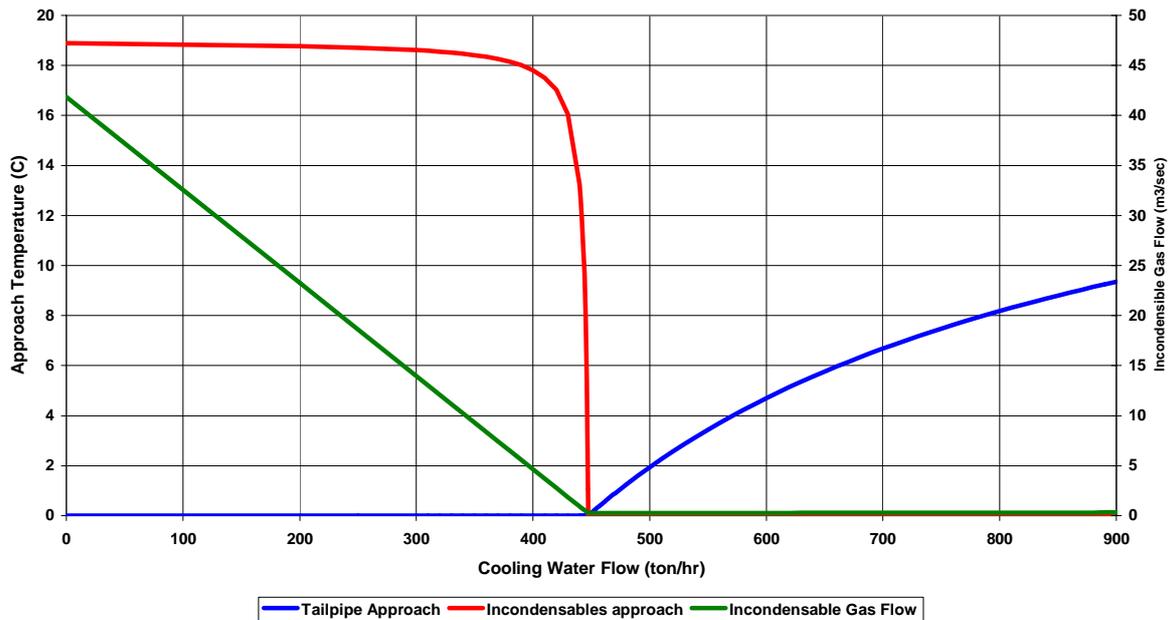
**Figure 4. Performance of ideal condenser operating at 15 kPa
with unlimited vacuum pump capacity.**

The two possible operating conditions are clearly evident. At low cooling water flows (below 447 ton/h) the tailpipe approach temperature is zero whilst the incondensables approach temperature is high, dropping rapidly to zero as the cooling water flow approaches the critical water flow of 447 ton/h where the maximum condensation takes place. At high cooling water flow rates (above 447 ton/h) the incondensables approach temperature drops to zero whilst the tailpipe approach temperature rises steadily as the cooling water flow increases.

Below the critical water flow rate the incondensable gas flow rate decreases steadily as the cooling water flow increases, due to the increase in condensation that takes place. Above the critical water flow rate no more condensation is possible as the incondensable gas cannot be cooled any further. Although it is not clearly evident in the graph (because of the scale of the Y axis) there is an increase in the flow of incondensable gas as the cooling water flow is increased above the critical flow. This is as a result of the increased quantity of air introduced with the cooling water.

Behaviour of an ideal vacuum pump

A liquid ring vacuum pump approximates a positive displacement pump although less so than the older type of piston pump which was once common in the sugar industry. A positive displacement pump will remove a constant volumetric flow, regardless of its pressure temperature and composition.

The ideal vacuum pump is thus taken as a positive displacement pump with these characteristics.

Behaviour of an ideal vacuum system

An ideal vacuum system is the combination of an ideal condenser with an ideal vacuum pump. With an ideal system (rather than just an ideal condenser) it is no longer possible to define both the operating pressure and the cooling water flow rate. Rather, we are limited to

defining the flow rate and composition of the vapour entering the condenser and the cooling water flow rate. The ideal condenser model must then be iterated to find the operating pressure at which the volumetric flow rate of incondensable gas matches the capacity of the ideal vacuum pump. The values used in this simulation (based on the previous simulation) are as follows:

Total mass flow of vapour into the condenser	15 ton/h
Concentration of air in vapour entering the condenser	3000 ppm
Concentration of air in cooling water	35 ppm
Cooling water temperature	35ºC
Volumetric capacity of vacuum pump	0.35 m^3/s

The results of the simulation are presented in Figure 5 and show the operating pressure in the condenser varies with cooling water flow rate. Actual incondensable gas temperature and tailpipe temperature are plotted rather than the approach temperatures plotted in the previous graph.
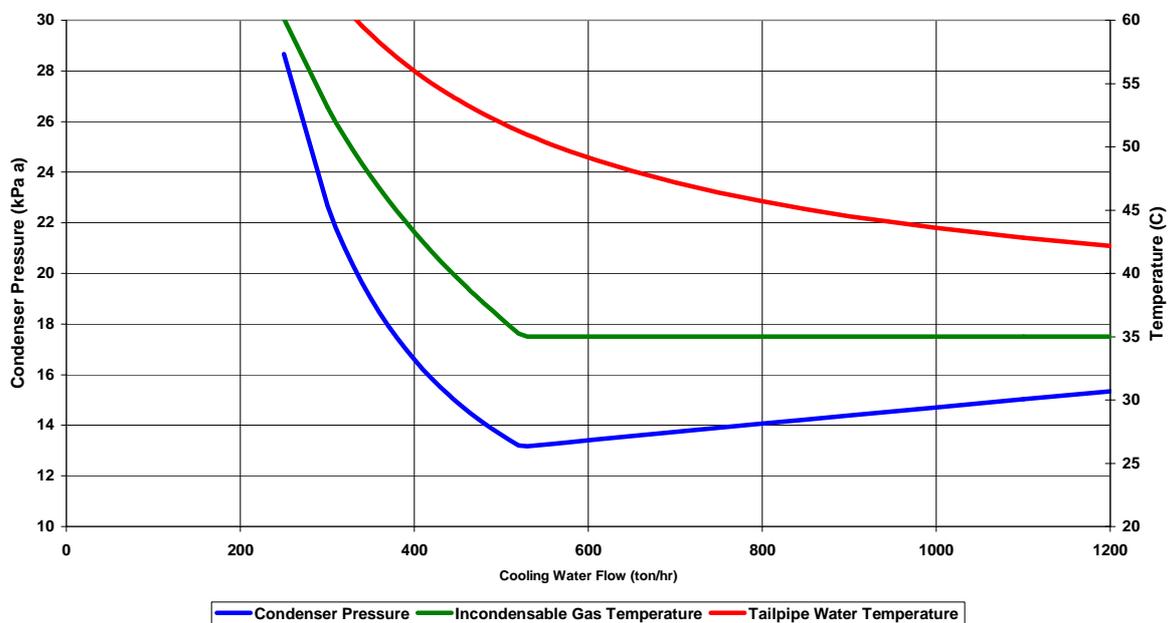

Figure 5. Performance of an ideal vacuum system.

The critical cooling water flow rate occurs at approximately 525 tons/h. At this flow, the incondensable gas is fully cooled (to 35ºC) and with higher cooling water flow rates no further cooling is possible. Below the critical water flow, increasing cooling water flow decreases the mass flow of vapour in the incondensable gas as a result of condensing more vapour. As a result, to keep the volumetric flow of incondensable gas constant, the pressure must drop. Above the critical cooling water flow, no further reduction in the quantity of vapour in the incondensable gas is possible. However the quantity of air to be removed in the incondensable gas continues to increases as a result of that entering with the cooling water. Thus above the critical cooling flow rate, increasing cooling water flow causes the pressure in the condenser to rise so as to compensate for the increasing mass flow in the incondensable gas stream.

Modelling real vacuum systems

Real vacuum systems deviate from the ideal systems presented above in a number of ways. For example:

- Cooling water is not normally set directly, it is the consequence of the degree of opening of a control valve. The relationship between control valve opening and cooling water flow is influenced by the characteristics of the control valve, other resistance in the supply line (e.g. a throttled manual valve) and the influence of the cooling water flow on the supply pressure.
- The condenser will not achieve the ideal of perfect counter current contacting. The deviation from ideal behaviour can be modelled by allowing a portion of the cooling water stream to bypass the condenser and flow into the tailpipe without any contact with the vapour. The magnitude of this bypass stream will be dependent in some way on the operating conditions of the condenser and it is this dependence which will describe the behaviour of any real condenser.
- The incondensable gas line will introduce a pressure drop between the condenser and the inlet of the vacuum pump. This can have a significant effect on system performance at high incondensable gas flows.
- Although liquid ring vacuum pumps do approximate positive displacement pumps when pumping dry air at moderate vacuum under sugar factory operating conditions there are a number of factors which cause deviations from this form of ideal behaviour (some increasing capacity and some decreasing capacity). These factors include vaporisation of the sealing water inside the pump, condensation of water vapour entering the pump into the sealing water inside the pump and pressure drops through the inlet ports of the pump (particularly when there is a substantial portion of water vapour in the incondensable gas supplied to the pump). Vacuum pump manufacturers usually supply extensive data on the magnitude of these deviations from ideal behaviour along with details of the performance of any particular pump model.
- There are recommendations that liquid ring vacuum pumps should operate with water vapour concentrations in the inlet gas below a specified maximum so to as to prevent cavitation and consequent damage to the pump.

It should be possible to include most of these effects into a more realistic (and more complex) model of a process vacuum system.

Use of the model to understand real system behaviour

Although the models presented in this paper only represent ideal systems they are still useful in helping to understand the behaviour of real systems. Sometimes, the relative simplicity of an ideal model is an advantage because it avoids the obfuscating complexity of a comprehensive model which accommodates a range of secondary and tertiary effects. The two examples described below, show how the ideal models can be used to make sense of the behaviour of real vacuum systems which at first seemed rather strange.

- *The 'frozen' condenser*
  Plant operators sometimes report an operating condition which they refer to as a 'frozen' condenser. The characteristics of this condition are that the tailpipe temperature is relatively cold, the incondensable gas temperature is cold, the control system has driven the cooling water valve full open but the vacuum system is unable to achieve the required operating pressure (the actual pressure is too high). The operators claim that this

condition can be cured by manually throttling the cooling water flow until the tailpipe and the incondensable gas line warm up (this is the 'unfreezing' of the condenser) and then slowly opening up the throttle valve, after which the automatic control system will take over control again and this time will be able to achieve the required operating pressure.

Whilst this scenario may at first seem incredible, it is explainable in terms of the results of the example shown in Figure 5. If we assume that the system modelled in this example is intended to operate at 14 kPa absolute, it would normally be operating at a cooling water flow rate of approximately 480 tons/h. Around this operating point, an increase in cooling water flow would cause the pressure to drop, and a decrease in cooling water flow would cause the pressure to rise – as expected and programmed into the action of the controller. Some short term instability (e.g. a fluctuation in water pressure caused by starting an extra cooling water pump) could easily cause a large increase in cooling water flow to a value above 800 tons/h. At this flow, the resulting pressure would be higher than the desired 14 kPa absolute and the controller would try to compensate by opening the control valve to increase the cooling water flow. This would only make the pressure worse; causing the controller to drive the cooling water valve further open until the valve is eventually fully open and the pressure has risen still further. At this point the tailpipe water temperature will be relatively low and the incondensable gasses will be cooled to their maximum. By using the manual valve on the cooling water line it is possible to throttle the water flow to below 400 ton/h, where both the tailpipe temperature and the incondensable gas temperature will increase. If at this point the throttle valve is opened slowly, as the cooling water increases beyond 480 tons/h the pressure will drop below 14 kPa absolute and the automatic control system will again take over control, throttling the flow to 480 tons/h to prevent the pressure from dropping below 14 kPa absolute.

- *Unexpected carryover of cooling water into the vacuum pump*
  While commissioning a newly installed continuous pan, problems were experienced with slugs of cooling water being carried over into the incondensable gas line. The carryover could be detected by noise generated by the vacuum pump and the burning rubber smell from the belts driving the pump (the overheated belts eventually snapped). The level of noise from the pump and the forces necessary to burn out the rubber belts indicated that major damage to the pump was inevitable if the problem was not solved quickly.

  The obvious 'suspect' was a problem with the design of water distribution system in the top of the condenser causing carryover of water into the incondensable gas line at high cooling water flows. Closer observation of the problem yielded surprising results. With the cooling water valve fully open, there was no carryover of water into the vacuum pump. The carryover only occurred when the cooling water was well throttled and the incondensable gas pipe was far too hot to touch. Using the results presented in Figure 4 as a guide it is possible to develop an explanation of why the system was behaving in this way. The vacuum pump had excess capacity. The vacuum pump would have been sized for the worst case conditions of maximum evaporation rate, maximum cooling water temperature and a relatively leaky pan. The operating conditions during commissioning were well away from the worst case. In particular, a vacuum test on the pan had shown it to be exceptionally leak tight. Under these circumstances, the load of air on the vacuum pump would be low and to increase the total gas flow up to the capacity of the pump, the cooling water would have to be reduced, resulting in an increase in the incondensable gas temperature. Although not addressed in the ideal condenser model, the higher proportion of water vapour in the incondensable gas would increase the capacity of the pump (due to condensation within the pump) resulting in a further increase in incondensable gas flow to

the pump. It was these very high gas flow rates within the top of the condenser which were resulting in the entrainment of cooling water into the incondensable gas line and causing the problems with the vacuum pump.

The short term solution was to open a bleed valve on the incondensable gas line, loading the vacuum pump with more air and thus moving the normal operating conditions of the condenser so that it now operated with a higher cooling water flow, colder incondensable gas temperature and a lower incondensable gas flow out of the top of the condenser. This solved the problem of water carryover and pump damage.

Conclusions

Simple models of a direct contact condenser and a vacuum pump which are based on fundamental physical chemistry are able to describe the behaviour of vacuum systems used in sugar factories. These models provide a very useful insight into the behaviour of real systems and can give useful guidance in solving operational problems. Extending these models by incorporating quantifications of the non-ideal behaviour of real systems would improve the usefulness of this approach.

REFERENCES

Toon ER, Ellis GE and Bodkin J (1968). *Foundations of Chemistry.* Holt, Rinehart and Winston, New York, USA.

NOMENCLATURE

The total flow of a stream in (kg/s) is defined by a single capital letter:

        C   for cooling water flow
        V   for flow of vapour leaving the process vessel
        I   for incondensable gas flow leaving the condenser
        B   for tailpipe water
        L   for the quantity of vapour condensed

Taking the incondensable gas stream as an example, the properties relating to a particular stream are named as follows:

        Ia   mass flow of air in kg/s
        Iw   mass flow of water (water vapour in this instance) in kg/s
        Ti   Temperature in ºC
        FIw   mass fraction of water
        FIa   mass fraction of air
        fIw   mole fraction of water
        fIa   mole fraction of air
        PI   Total pressure in kPa absolute
                    (*PI = PV = P* with constant pressure in the condenser)
        PIw   Partial pressure of water vapour in kPa absolute
        PIa   Partial pressure of air in kPa absolute
        HIw   Enthalpy of water vapour in kJ/kg, where *HIw = H(Ti)*

Because the gaseous streams are assumed to be at saturation (i.e. water vapour is in equilibrium with liquid water at the same pressure and temperature), this situation can be expressed mathematically as follows:

        PIw   =   *Ps(Ti)*
        Ti   =   *Ts(PIw)*
        HIw   =   *H(Ti)*

Where:

        Ps(t)   Is an empirical equation expressing the saturated vapour pressure of steam as a function of temperature, *t*.
        Ts(p)   Is an empirical equation expressing the saturated vapour temperature of steam as a function of the pressure, *p*.
        H(t)   Is an empirical equation expressing the enthalpy of saturated steam at a temperature, *t*.

Also:

SHa   is the specific heat of air in kJ/(kg ºC)
SHw is the specific heat of water (in the liquid phase) in kJ/(kg ºC)

APPENDIX 1
Fundamentals of Gas Behaviour

An understanding of behaviour under changing conditions of temperature, pressure quantity and composition requires a knowledge of a number of fundamental relationships which describe gas behaviour (Toon et al, 1968). These can be summarised as follows:

Boyle's Law: 'The volume of a fixed mass of gas varies inversely with the pressure at constant temperature.'

Charles' Law: 'The volume of a given mass of gas at constant pressure is directly proportional to its absolute temperature.'

Avagadro's Law: 'Equal volumes of gasses under the same conditions of temperature and pressure contain the same number of molecules (or moles).'

The Ideal Gas Equation (which encompasses the laws of Boyle, Charles and Avagadro):

$$P \cdot V = n \cdot R \cdot T$$
Where P is the pressure of the gas (in absolute, and not gauge, units)
T is the temperature of the gas (in absolute temperature units)
V is the volume of the gas
n is the number of moles of gas
R is the universal gas constant

Rather than use the value of R, it is often convenient to use the simplification that one mole of gas occupies a volume of 22.4 litres at a temperature of 101.3 kPa absolute and 0ºC.

Dalton's Law of Partial Pressures: 'The total pressure exerted by a gaseous mixture at constant temperature is equal to the sum of the pressures that each of the gasses composing the mixture would exert if each occupied the same volume alone.'

Combining Dalton's Law with the Ideal Gas Equation yields the relationship:
Partial pressure = mole fraction · total pressure

In the modelling of vacuum system behaviour we will consider all gaseous streams to consist of only two components viz. air (with a molecular mass $ma = 29$) and water vapour (with a molecular mass $mw = 18$).

Steam tables: Steam tables provide extensive numeric data on the behaviour of water in both the liquid and gaseous phases. In particular they provide data on saturated steam (steam, or water vapour, which is in equilibrium with water at the same temperature). For saturated steam, there is a direct relationship between temperature and pressure. Empirical equations are available to define this relationship enabling us to write:

$$Pw = Ps(T)$$
and $T = Ts(Pw)$

Where Pw is the partial pressure of water vapour
$Ps(T)$ is an empirical equation which calculates the saturated vapour pressure from the temperature T

Ts(*Pw*) is an empirical equation which calculates the saturated vapour temperature from the partial pressure of the water vapour *Pw*

Empirical equations are also available to calculate the enthalpy of saturated steam (water vapour), enabling us to express:

$$H = H(T)$$

For air and liquid water it is convenient to calculate the heat content (enthalpy) of a stream from the specific heat, using the terminology:

SHa is the specific heat of air
SHw is the specific heat of liquid water.

For a gaseous stream of total flow *X*, air flow *Xa* and water vapour flow *Xw*, it is possible to write the following equations to calculate the mass fraction of air, *FXa*, the mass fraction of water, *FXw*, the mole fraction of air, *fXa*, and the mole fraction of water, *fXw*.

$$FXa = \frac{1}{1 + \dfrac{ma}{mw} \cdot \left(\dfrac{1}{fXa} - 1 \right)}$$

$$FXw = \frac{1}{1 + \dfrac{mw}{ma} \cdot \left(\dfrac{1}{fXw} - 1 \right)}$$

$$fXa = \frac{FXa \cdot ma}{mw \cdot (1 - FXa) + FXa \cdot ma}$$

$$fXw = \frac{FXw \cdot mw}{ma \cdot (1 - FXw) + FXw \cdot mw}$$

APPENDIX 2
Equations for Model of an Ideal Condenser

First calculate the quantity of air in the incondensable gas stream from that entering with the vapour and with the cooling water:

$$Ia = Va + Ca$$

The condenser must operate at one of two possible limiting conditions, either a 'high cooling water flow' condition where the incondensable gas stream is cooled down to the cooling water temperature or the 'low cooling water flow' condition where the tailpipe water leaves at the temperature of the vapour entering the condenser. The calculation procedures are different for each of these conditions.

- *High cooling water flow*
  For this option, fix the temperature of the incondensable gas equal to the cooling water temperature.
  $Ti = Tc$
  Calculate the partial pressure of water vapour in the incondensable gas stream
  $PIw = Ps(Ti)$
  Using this, determine the mole fraction of water vapour in the incondensable gas stream
  $$fIw = \frac{PIw}{PI}$$
  Calculate the mass fraction of water vapour in the incondensable gas stream
  $$FIw = \frac{1}{1 + \dfrac{mw}{ma} \cdot \left(\dfrac{1}{fIw} - 1 \right)}$$
  With the knowledge of the flow of air in the incondensable gas stream, it is possible to calculate the total flow of the incondensable gas stream:
  $$I = \frac{Ia}{1 - FIw}$$
  And then the water flow in the incondensable gas stream:
  $Iw = I - Ia$
  The quantity of water vapour condensed is then given by the difference between the quantity entering in the vapour and that leaving with the incondensable gas stream:
  $L = Vw - Iw$
  The tailpipe water flow is given by the sum of the flow of water in the cooling water stream and the quantity of vapour condensed:
  $B = Cw + L$ (note that there is no air in the tailpipe water stream)
  To determine the tailpipe water temperature, it is necessary to set up a heat balance over the whole condenser as follows:
  Heat Input with Cooling Water (both water and air components)
  $Ca \cdot SHa \cdot Tc + Cw \cdot SHw \cdot Tc$
  Heat Input with Vapour from Vessel (both water and air components
  $Va \cdot SHa \cdot Tv + Vw \cdot H(Tv)$
  Heat Output with Incondensable Gas Stream (both water and air components)
  $Ia \cdot SHa \cdot Ti + Iw \cdot SHw \cdot Ti$
  Heat output in Tailpipe Water (there is only water in this stream)
  $B \cdot SHw \cdot Tb$

Using these four expressions, total heat input can be equated to total heat output to yield:

$$(Ca \cdot SHa + Cw \cdot SHw) \cdot Tc + Va \cdot SHa \cdot Tv + Vw \cdot H(Tv) = Ia \cdot SHa \cdot Ti + Iw \cdot SHw \cdot Ti + B \cdot SHw \cdot Tb$$

This expression can then be solved to give:

$$Tb = \frac{(Ca \cdot SHa + Cw \cdot SHw) \cdot Tc + Va \cdot SHa \cdot Tv + Vw \cdot H(Tv) - (Ia \cdot SHa + Iw \cdot SHw) \cdot Ti}{B \cdot SHw}$$

If the temperature, Tb, is greater than the temperature of the entering vapour, Tv, then this is not a feasible operating condition.

- *Low cooling water flow*
  At low cooling water flows, the tailpipe water temperature is assumed to be equal to the incoming vapour temperature i.e.

$$Tb = Tv$$

Since the temperature of the incondensable gas is not known at this stage, it is necessary to set up a heat balance over the whole condenser to determine this. The heat balance equations below have been formulated to include the quantity of vapour condensed (L) because this simplifies the calculation of the incondensable gas stream.

Heat Input with Cooling Water (both water and air components):
$Ca \cdot SHa \cdot Tc + Cw \cdot SHw \cdot Tc$
Heat Input with Air in Vapour from Vessel:
$Va \cdot SHa \cdot Tv$
Heat Input with Water in Vapour from Vessel:
$L \cdot H(Tv) + (Vw - L) \cdot H(Tv)$
Heat Output with Air in Incondensable Gas Stream:
$(Ca + Va) \cdot SHa \cdot Ti$
Heat Output with Water in Incondensable Gas Stream:
$(Vw - L) \cdot H(Ti)$
Heat output in Tailpipe Water (there is only water in this stream):
$(L + Cw) \cdot SHw \cdot Tb$

The heat balance is set up by using these equations to equate the total heat flow into the condenser to the total heat flow out of the condenser as follows:

$$Ca \cdot SHa \cdot Tc + Cw \cdot SHw \cdot Tc + Va \cdot SHa \cdot Tv + L \cdot H(Tv) + (Vw - L) \cdot H(Tv)$$
$$= (Ca + Va) \cdot SHa \cdot Ti + (Vw - L) \cdot H(Ti) + (L + Cw) \cdot SHw \cdot Tb$$

There are two unknowns in this resulting equation, viz. L and Ti. Fortunately (by using a conceptual understanding of the processes taking place within the condenser) it is possible to simplify the equation in such a way as to develop an equation for calculating L which is only slightly dependent on the (as yet unknown) details of the incondensable gas stream. This is done by using the relationship:

$$Iw = Vw - L$$

to yield:

$$L = \frac{Cw \cdot SHw \cdot (Tb - Tc) - Iw \cdot (H(Tv) - H(Ti)) - Va \cdot SHa \cdot (Tv - Ti) + Ca \cdot SHa \cdot (Ti - Tc)}{H(Tv) - SHw \cdot Tb}$$

The numerator of the equation can be interpreted as

$$H0 - H1 - H2 + H3$$

Where :
$H0$ is the heat acquired by the water in the cooling water stream as it is heated from its starting temperature up to the tailpipe water temperature.
$H1$ is the heat lost by the uncondensed vapour as it is cooled from its starting temperature down to the incondensable gas temperature.
$H2$ is the heat lost by the air which enters in the vapour stream as it is cooled from its starting temperature down to the incondensable gas temperature.
$H3$ is the heat acquired by the air entering with the cooling water as it is heated from its starting temperature up to the incondensable gas temperature.

Figures obtained by simulating realistic conditions show that the values of $H1$, $H2$ and $H3$ are of the order of only 0.01% of the value of $H0$. A reasonably accurate initial estimate of the value of L can thus be made by neglecting the terms $H1$, $H2$ and $H3$. Using this estimate of L, it is then possible to calculate the properties of the incondensable gas stream, specifically Iw and Ti. With these values, the value of L can be recalculated without neglecting the terms $H1$, $H2$ and $H3$. This process can then be repeated until convergence is achieved (a task easily accomplished by the use of circular references in a spreadsheet).

If the value of L determined in this way is greater than that calculated for a fully cooled incondensable gas stream (the conditions assumed for the 'high cooling water flow' condition which define the maximum quantity of condensation which can be achieved) then this is not a feasible operating condition.

For a feasible estimate of the quantity of vapour condensed, the calculation of Iw and Ti proceeds as follows:
$Iw = Vw - L$
The mass fraction of water vapour in the incondensable gas stream is thus:
$$FIw = \frac{Iw}{Iw + Ia}$$
This can then be converted into a mole fraction of water vapour
$$fIw = \frac{FIw \cdot mw}{ma \cdot (1 - FIw) + FIw \cdot mw}$$
From which the partial pressure of water vapour can be calculated
$PIw = P \cdot fIw$
This can then be used to calculate the saturated vapour temperature, which is the Incondensable Gas temperature:
$Ti = Ts(PIw)$