OPTIMUM DISTRIBUTION OF HEATING SURFACE IN A MULTIPLE EFFECT EVAPORATOR TRAIN

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Abstract

The overall specific evaporation rate (in kg/h/m^2) of a multiple effect evaporator train is highly dependent on the distribution of heating surface among the effects. Previous authors (Hugot; Buczolich and Zadori) have each given criteria for achieving optimum heating surface distributions. The criteria given by the above authors are different from each other, and while they get very close to an optimum distribution, better distributions can be achieved.

The present work uses a spreadsheet model of a multiple effect evaporator train and the optimising routines in the spreadsheet software to find the distribution of heating surface along the evaporator train which gives the highest specific evaporation rate. The vapour temperatures are the parameters that are varied in order to maximise the overall specific evaporation rate.

Similarities between the previous authors' criteria and the results of the spreadsheet optimisation are discussed.

Keywords: evaporator, evaporators, multiple effect evaporator, modelling, performance, factory process

Introduction

Background

The overall specific evaporation (in kg/h/m^2) of a multiple effect evaporator train is highly dependant on the distribution of heating surface among the effects. Previous authors have given criteria for achieving an optimum distribution. The criteria given by the previous authors are different from each other, and while they get very close to an optimum distribution, better distributions can be achieved.

Buczolich and Zadori (1963) state that when there is an "..optimal distribution of heating surfaces, the surfaces of the single stages are in proportion to the correspondent temperature drop in the single stages." They express this relation mathematically as:

$$A_1 : A_2 : \ldots : A_n = \Delta T_1 : \Delta T_2 : \ldots : \Delta T_n \qquad (1)$$

This can also be stated as:

$$\frac{A_i}{\Delta T_i} = C \qquad (2)$$

where C is a constant.

In arriving at this criterion, Buczolich and Zadori start first with a twin effect evaporator train and mathematically find the optimum ratio of area for the two effects (by taking the derivative of the function relating ΔT and A and setting it to zero and then solving). The relation for two effects is then assumed to apply to any two consecutive effects of any multiple effect evaporator.

Hugot (1972) states, "To obtain a minimal heating surface for the multiple effect, the ratio of the heating surface of a vessel to the sum of the heating surfaces of the following vessels is twice the ratio of the temperature drop for that vessel to the sum of the temperature drops of the following vessels." In mathematical notation this is stated as:

$$\frac{A_i}{\sum_{j=i+1}^{n} A_j} = 2 \cdot \frac{\Delta T_i}{\sum_{j=i+1}^{n} \Delta T_j} \tag{3}$$

This equation may be rearranged as follows:

$$\frac{A_i}{\Delta T_i} = 2 \cdot \frac{\sum_{j=i+1}^{n} A_j}{\sum_{j=i+1}^{n} \Delta T_j} \tag{4}$$

Hugot calculates the optimum heating surface for the first effect relative to the sum of the other effects assuming that the distribution of heating surfaces of the other effects is already optimum.

While the left hand sides of equations 2 and 4 are identical the right hand sides are very different and cannot be satisfied simultaneously, implying one or both of these criteria do not give a true optimum.

Theory
By definition:

$$_JT_i = {_vT_i} + BPE_i \tag{5}$$

Boiling point elevation (BPE) is a function of purity, dry substance, temperature and hydrostatic head (Bubnik *et al.,* 1995). The driving force temperature difference for each effect is given by:

$$\Delta T_i = {_vT_{i-1}} - {_JT_i} \tag{6}$$

Now the heat transferred across each heat transfer surface is:

$$Q_i = k_i \cdot A_i \cdot \Delta T_i \tag{7}$$

Also

$$Q_i = {_vm_i} \cdot \Delta h_i \tag{8}$$

where Δh_i is the latent heat of vaporisation of water in effect *i*. The values of latent heat are functions of the vapour temperatures in each effect.

Combining the above two equations and rearranging gives:

$$\frac{_v m_i}{A_i} = \frac{k_i \cdot \Delta T_i}{\Delta h_i} \tag{9}$$

The term on the left hand side of the above equation is the specific evaporation rate, and can be called SE_i.

The total evaporation of the evaporator train

$$_v m_{tot} = \sum_{i=1}^{n} {}_v m_i \tag{10}$$

is calculated by the difference between the water in clear juice and the water in syrup, and that calculation can be reduced to:

$$_v m_{tot} = {}_J m_0 \left(1 - \frac{_J b_0}{_J b_n} \right) \tag{11}$$

where the subscript 0 refers to flow *into* the first effect.

The mass of vapour leaving the final effect is given by:

$$_v m_n = \frac{_v m_{tot} - \sum_{i=1}^{n-1} z_i \cdot B_i}{n} \tag{12}$$

where:

$$z_i = \Delta h_i \sum_{j=1}^{i} \frac{1}{\Delta h_j} \tag{13}$$

Equation 13 is strictly correct but, if one assumes $\Delta h_1 = \Delta h_2 = \Delta h_3 = \ldots = \Delta h_n = \Delta h$ then the approximation $z_i \approx i$ may be used.

The evaporation from the other effects is given by:

$$_v m_i = {}_v m_{i+1} + B_i \quad \text{where } i = 1 \text{ to } n\text{-}1 \tag{14}$$

Finally we can calculate the required heat transfer area for each effect as follows:

$$A_i = \frac{_v m_i}{SE_i} \tag{15}$$

The total heat transfer area is:

$$A_{tot} = \sum_{i=1}^{n} A_i \tag{16}$$

The overall specific evaporation is then:

$$SE = \frac{\sum_{i=1}^{n} {}_v m_i}{\sum_{i=1}^{n} A_i}$$ (17)

If this parameter is maximised then the heating surface will be optimally distributed. It is assumed that parameters such as juice flow, juice brix, syrup brix, exhaust steam temperature, and final effect vacuum are fixed. The only parameters that may be changed are the temperatures of the vapours. The temperature of the heating steam supplied to the first effect and the temperature of the vapour leaving the last effect are not varied.

If bled vapours are used for juice heating and pan boiling it is important to ensure that the bled vapours are maintained at temperatures sufficient to provide an adequate temperature difference across the heating surfaces of the pans and heaters.

Procedures

Microsoft Excel spreadsheets were constructed using the equations above to model a five-effect evaporator train: print outs are shown in Appendix 2, 3, and 4.

These models were used to calculate the heat transfer surface areas of each effect:
- for a non-optimised evaporator train
- for an evaporator train optimised according to Hugot's criterion (equation 4 above)
- for an evaporator train optimised according to the criterion of Buczolich and Zadori (equation 2 above)
- and for an evaporator train optimised by means of the Microsoft Excel 'Solver'. While one cannot be mathematically certain that this numerically calculated solution is a true optimum, it is assumed that is very close to the true optimum and will be referred to as the *optimum* solution hereafter.

Overall heat transfer coefficients
There are a number of choices for how one calculates the overall heat transfer coefficient (OHTC) k_i. Hugot assumed the OHTC is governed by the Dessin formula:

$$c_i = C_D \cdot (100 - {}_J b_i) \cdot ({}_J T_i - 54)$$ (18)

Where C_D is a constant, Hugot uses the value 0.001. The OHTC can be calculated by multiplying the Dessin evaporation coefficient by the latent heat of vaporisation of water in effect i, that is:

$$k_i = C_D \cdot (100 - {}_J b_i) \cdot ({}_J T_i - 54) \cdot \Delta h_i$$ (19)

So in Hugot's analysis the OHTC is assumed to be a function juice brix and temperature.

Buczolich and Zadori's considered the OHTC to be a function of the evaporator effect number only; the values they used are shown in Table 1 below. In their paper on a *robust* design of an evaporator station, Love, Meadows and Hoekstra (1999) use a similar approach for values of OHTC; their values are also shown in Table 1.

Table 1. Overall heat transfer coefficients.

Effect	Buczolich and Zadori		Love, Meadows and Hoekstra
	kcal/m²/h/°C	kW/m²/K	kW/m²/K
1	3000	3.489	2.500
2	2000	2.326	2.500
3	1250	1.454	2.000
4	~	~	1.500
5	~	~	0.700

A third option is to use the function given by Urbaniec in Van der Poel *et al.* (1998):

$$k_i = C_U \cdot \frac{_J T_i}{_J b_i} \tag{20}$$

where C_U is a constant. The present author uses this approach. Smith and Taylor (1981) suggest, from factory data, that OHTC is a function of brix and juice temperature, they go on to add, "There is little evidence of reduction in [heat transfer coefficient] HTC with increasing effect number."

In order to reduce the effect of differences in assumptions about OHTC the following steps were taken: In the case of the Buczolich and Zadori optimisation the OHTCs were assumed to have the following fixed values:

Effect	OHTC [kW/m²/K]
1	2.480
2	1.960
3	1.440
4	0.920
5	0.400

The temperatures in each effect were adjusted until the Buczolich and Zadori optimisation criterion was met, that is all $A_i / \Delta T_i = C$. In the case of the Hugot optimisation, the Dessin constant C_D and the temperatures in each effect were adjusted so that the total heat transfer area was equal to that calculated by the Buczolich and Zadori optimisation, and the Hugot optimisation criterion was met, (see equation 4). For the final case, the Urbaniec constant C_U was adjusted so that the total heat transfer area was equal to that calculated by the Buczolich and Zadori optimisation and furthermore the temperatures in each effect were adjusted so that the specific evaporation rate was maximised.

Input parameters
The calculations were done using the parameters given in Table 2.

Table 2. Input parameters.

Clear Juice Flow [ton/h]	500
Clear Juice Purity	85.0%
Clear Juice Brix	13.5%
Syrup Brix	65.0%
Heating Steam Temperature [ºC]	124
Final Effect Vapour Temperature [ºC]	65

In order to compare the three optimisation methods it was decided *not* to constrain the bleed vapour temperatures, because neither Buczolich and Zadori nor Hugot incorporated a bleed vapour temperature constraint in their analyses. Vapour bleed was modelled as follows: V1 and V2 for juice heating and pan boiling, in a way that was consistent for all four scenarios; that is to say the heat load was the same for all four scenarios. The actual mass flow of bled vapours differed because the steam temperatures and hence enthalpies were different.

The non-optimised scenario was calculated using a linear temperature profile as shown in Figure 1.
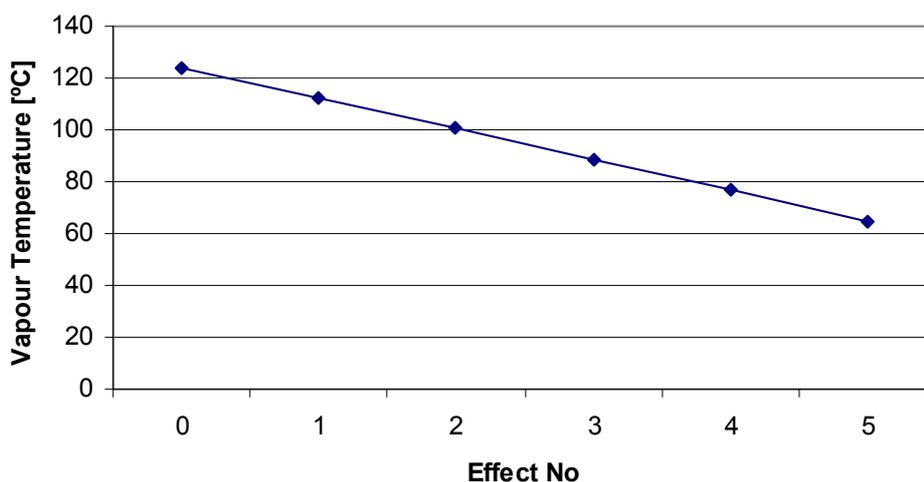
Figure 1. Temperature profile.

For the other scenarios the temperature profile was adjusted so that:
- Hugot's criterion was met
- the criterion of Buczolich and Zadori was met, or
- the specific evaporation rate was maximised.

Results and discussion

Results
The heat transfer areas calculated are given in Table 3 and also shown graphically in Figure 2.

Table 3. Calculated heat transfer areas.

|	Heat Transfer Area [m²]					
	1	2	3	4	5	Total
Non-optimised	3907.1	3592.9	1904.6	2909.3	5499.2	17813.1
Optimum	4221.8	3747.0	2499.1	2859.1	3446.0	16773.0
Buczolich and Zadori	3897.2	3248.5	2320.6	2903.3	4403.2	16773.0
Hugot	4838.2	3662.5	2332.9	2462.3	3477.1	16773.0
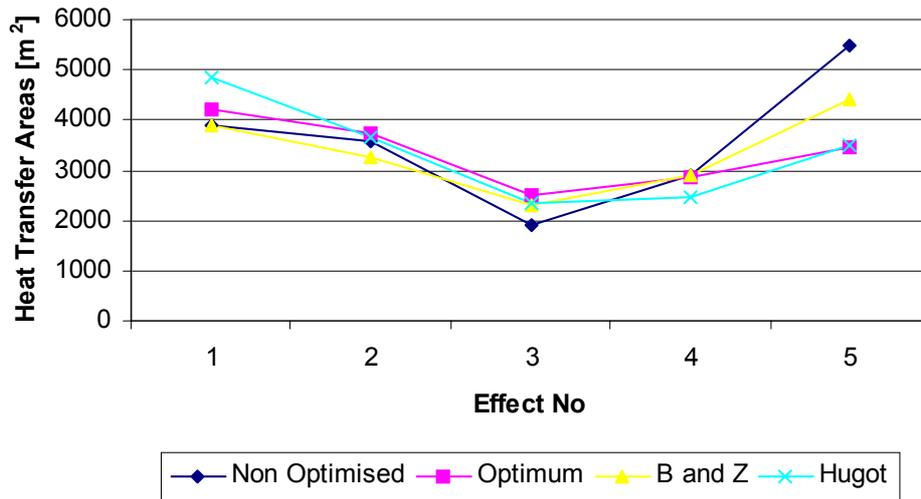
Figure 2. Heat transfer area profile.

It is clear from the table that the three optimised cases were calculated in such away that the OHTCs used gave equal total heat transfer area. As is expected the non-optimised case has a total heat transfer larger than the others. The OHTCs used in the various models are shown in Table 4 and again graphically in Figure 3.
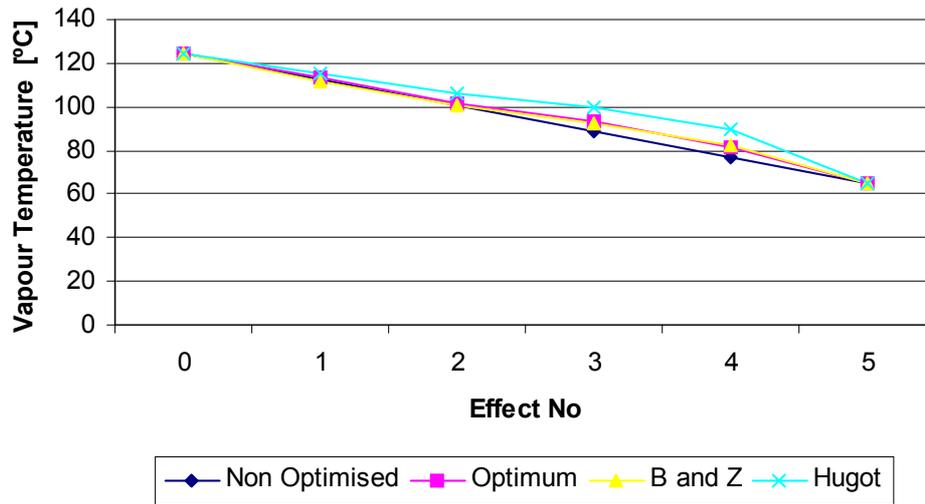
Table 4. Overall heat transfer coefficients.

|	OHTC [kW/m^2/K]				
	1	2	3	4	5
Non-optimised	2.602	1.595	1.168	0.810	0.524
Optimum	2.618	1.616	1.224	0.858	0.524
Buczolich and Zadori	2.480	1.960	1.440	0.920	0.400
Hugot	2.790	2.214	1.747	1.243	0.312
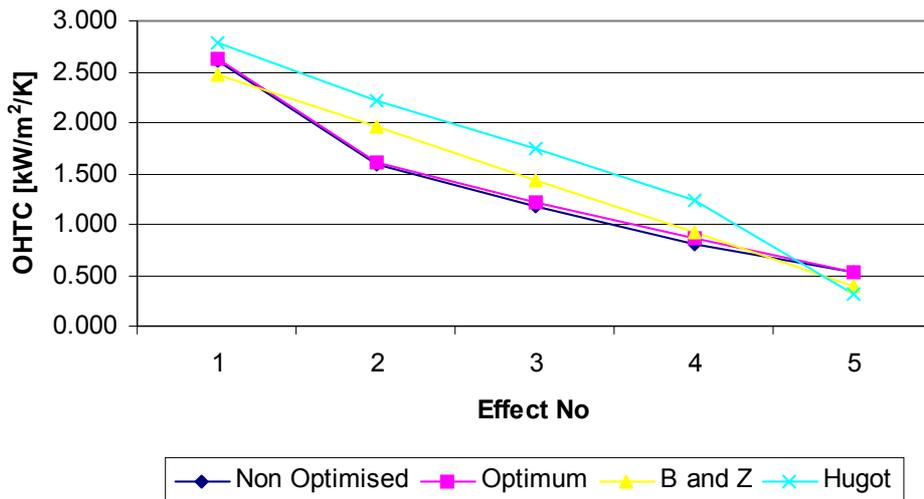
Figure 3. OHTC profile.

The temperature and temperature difference profiles for the various scenarios are shown in Figures 4 and 5.

Figure 4. Vapour temperature profile.

The Hugot optimisation (with the Dessin OHTCs) causes the vapour temperatures to be hotter compared to the other models. The consequence of this is the driving force temperature difference for the last effect is much higher for the Hugot optimisation than for others, as can be seen in Figure 5.
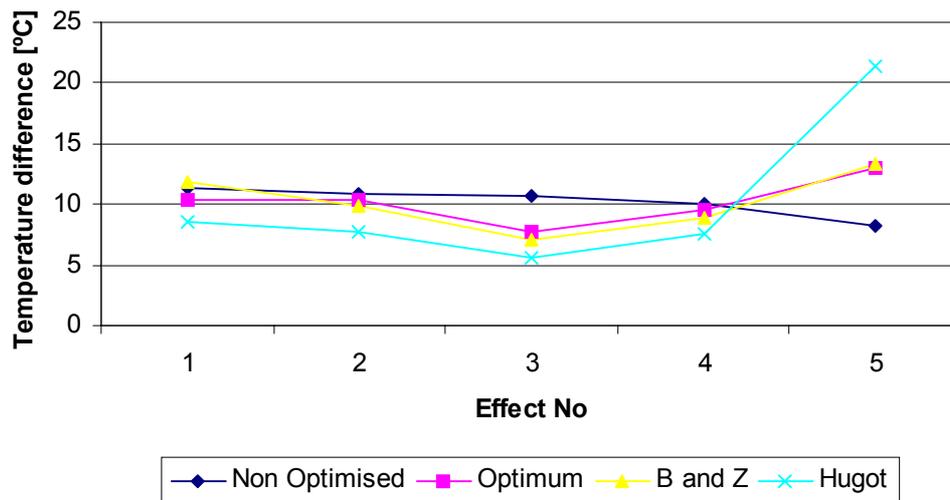
Figure 5. Driving force temperature difference profile.

It is interesting that when the heating surfaces are distributed optimally the driving force temperature difference profile has a 'U' shape. This is because, at the tail of the evaporator where the temperature is low and the viscosity is high, the OHTC is low; and in order to minimise the heat transfer area, the ΔT must be high. At the front end of the evaporator train when a large amount of vapour is required as bled vapour for other heating duties, the ΔT should be high to avoid large areas here.

Discussion
If one compares the heat transfer area distributions of the Hugot case on the one hand and the Buczolich and Zadori case on the other, one can see that Hugot gives a larger area for the first two effects, the third effects are about the same size, and Hugot's tail vessels are smaller than Buczolich and Zadori's. This is mostly because the Hugot model has higher OHTCs for the

early effects and a much larger driving force temperature difference in the last effect.

The values of $A_i/\Delta T_i$ are plotted against $\sum_{j=i}^{n} A_j \bigg/ \sum_{j=i}^{n} \Delta T_j$ in Figure 6.

In the non-optimised case one can see there is no particular relationship. In the optimised case this graph is a straight line (the correlation coefficient in this example is 0.99). The slope has a value of about 2.5, this value changes if the physical parameters such as steam temperature or juice brix and purity are changed. However there appears to be no obvious physical significance to the values of the slope and intercept of this straight line. The Buczolich and Zadori optimisation appears as a single dot in Figure 6; this is because all the $A_i/\Delta T_i = C$. In the Hugot optimisation the slope is exactly 2 and the y-intercept passes through the origin (confirming that equation 4 is satisfied)
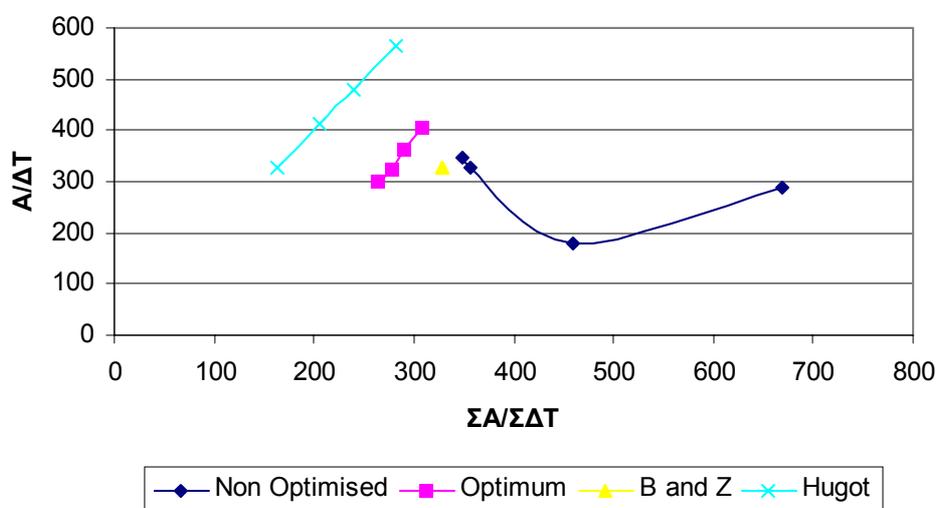
Figure 6. Ratios of heat transfer area and temperature difference.

Direct optimisation
Using the Solver function of MS Excel the specific evaporation rate was maximised (using the fixed OHTCs as in the Buczolich and Zadori analysis): the total area calculated was 16772.1 m^2, which is essentially the same value calculated by setting all $A_i/\Delta T_i = C$. This shows that assuming fixed OHTCs the Buczolich and Zadori optimisation gives the same result as the direct approach. If the same direct optimisation procedure is followed for the Hugot model using Dessin derived OHTCs the total calculated heat transfer area is a little less (16514.5 m^2 – about 1.5% difference), showing the Hugot criterion does not give a true optimum, although the solution it does yield is very close to the true optimum.

Bleed vapour temperatures
The calculated optimum distribution of heating surface may not be the ideal technological solution because the bleed vapour temperatures may be too low. The solver function of MS Excel can address this by setting a constraint. For example, the solver function can be asked to maximise the specific evaporation rate by varying the vapour temperatures, subject to the constraint that the V2 temperature is greater than say, 104ºC.

Conclusions

Buczolich and Zadori
If one accepts the assumptions about OHTC that Buczolich and Zadori make, that is, OHTC is a function of effect number, then their optimisation criterion gives a true optimum. However if one believes that OHTC is a function of juice temperature and brix, then the Buczolich and Zadori criterion (that all $A_i/\Delta T_i = C$) does not give a true optimum. The reason for this is the in the derivation of the Buczolich and Zadori criterion; they did a differentiation on the function relating ΔT and A, keeping k constant. It is clear that if one now varies k the result will not be correct.

Hugot
The Hugot criterion does not give a true optimum - the areas calculated by adjusting the vapour temperatures so that equation 4 is satisfied gives a larger total area than if the vapour temperatures are chosen so that the specific evaporation rate is maximised. In the derivation of his criterion Hugot makes a number of assumptions, namely:
- boiling point elevations are proportional to the nett temperature drops
- the basic temperature of the Dessin formula (54ºC) may be substituted for the temperature corresponding to the vacuum, and finally
- Hugot calculates the optimum heating surface for the first effect relative to the sum of the other effects assuming that the distribution of heating surfaces of the other effects is already optimum

These assumptions are sufficient to cause a 1.5% difference in the calculation of the optimum distribution of the area.

Direct optimisation
Spreadsheets (and specifically the solver function) can be used to calculate the required vapour temperatures and hence the heat transfer areas so that overall specific evaporation rate is maximised. In addition the solver function allows the setting of constraints to achieve a technologically acceptable solution. The application of the optimising criteria of Hugot and Buczolich and Zadori do not allow the easy computation of a solution with constraints.

Acknowledgements

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APPENDIX 1

Nomenclature

T Temperature [ºC]
b Brix [ºBx]
B Bleed mass flow [kg/h]
m Mass flow [kg/h]
h Specific enthalpy of water substance [kJ/kg]
A Heat transfer area [m^2]
SE Specific evaporation rate [$kg/m^2/h$]
k Overall heat transfer coefficient [$kW/m^2/K$]
c Dessin evaporation coefficient [$kg/h/m^2/ºC$]

Subscripts
J on the left of the variable refers to juice
v on the left of the variable refers to vapour
i and j on the right of the variable refer to the effect number
n is the total number of effects

APPENDIX 2

Spreadsheet printout - Optimum case

Input Parameters	
Clear Juice Flow (t/h)	500.0
Clear Juice Brix	13.50%
CJ Purity	85.00%
Syrup Brix	65.00%
Heat Transfer Factor	0.4964

Outputs	
Area	16773.0 m^2
Sp Evaporation	23.6 kg/m^2/h
Exh Steam Req	197.2 t/h
Brix rate	67.5 t/h
Total Evaporation	396.2 t/h

Bleed Requirements		
	t/h	Losses
V1	85.6	10%
V2	62.0	10%
V3	0.0	
V4	0.0	

Effect	Temperature (ºC)	Pressure (kPa)	Δh (kJ/kg)	BPE (ºC)	Juice temp (ºC)	k (kW/m^2/K)	ΔT (ºC)	Sp evap (kg/m^2/h)	Area (m^2)	Evaporation (t/h)	Juice flow (t/h)	Brix	Steam req (t/h)
0	124.0	224.0	2191.3								500.0	13.50%	
1	113.1	158.0	2221.9	0.50	113.6	2.618	10.42	44.2	4221.8	186.5	313.5	21.53%	189.1
2	101.9	107.8	2252.3	0.90	102.8	1.616	10.29	26.6	3747.0	99.6	213.8	31.57%	101.0
3	93.0	78.1	2275.8	1.20	94.2	1.224	7.69	14.9	2499.1	37.2	176.6	38.21%	37.6
4	81.6	50.2	2305.1	1.82	83.4	0.858	9.58	12.8	2859.1	36.7	139.9	48.24%	37.2
5	65.0	24.8	2346.7	3.58	68.6	0.524	13.03	10.5	3446.0	36.1	103.8	65.00%	36.7

APPENDIX 3

Spreadsheet printout - Buczolich and Zadori optimisation

Input Parameters	
Clear Juice Flow [t/h]	500.0
Clear Juice Brix	13.50%
CJ Purity	85.00%
Syrup Brix	65.00%
Heat Transfer Factor	

Outputs	
Area	16773.0 m^2
Sp Evaporation	23.6 kg/m^2/h
Exh Steam Req	196.0 t/h
Brix rate	67.5 t/h
Total Evaporation	396.2 t/h

Bleed Requirements		
	t/h	Losses
V1	83.6	10%
V2	62.8	10%
V3	0.0	
V4	0.0	

Effect	Temperature (ºC)	Pressure (kPa)	Δh (kJ/kg)	BPE (ºC)	Juice temp (ºC)	Fixed k (kW/m^2/K)	ΔT (ºC)	Sp evap (kg/m^2/h)	Area (m^2)	Evaporation (t/h)	Juice flow (t/h)	Brix	Steam req (t/h)
0	124.0	224.0	2191.3								500.0	13.50%	
1	111.6	150.6	2225.9	0.49	112.1	2.480	11.86	47.6	3897.2	185.3	314.7	21.45%	188.3
2	100.9	104.0	2255.0	0.89	101.8	1.960	9.88	30.9	3248.5	100.5	214.2	31.51%	101.8
3	92.6	77.0	2276.8	1.19	93.8	1.440	7.06	16.1	2320.6	37.3	176.9	38.16%	37.7
4	82.0	51.0	2304.2	1.82	83.8	0.920	8.83	12.7	2903.3	36.9	140.0	48.20%	37.3
5	65.0	24.8	2346.7	3.58	68.6	0.400	13.39	8.2	4403.2	36.2	103.8	65.00%	36.9

APPENDIX 4

Spreadsheet printout - Hugot optimisation

Input Parameters	
Clear Juice Flow (t/h)	500.0
Clear Juice Brix	13.50%
CJ Purity	85.00%
Syrup Brix	65.00%
Heat Transfer Factor	0.00107

Outputs	
Area	16773.0 m^2
Sp Evaporation	23.6 kg/m^2/h
Exh Steam Req	198.1 t/h
Brix rate	67.5 t/h
Total Evaporation	396.2 t/h

Bleed Requirements		
	t/h	Losses
V1	87.0	10%
V2	62.6	10%
V3	0.0	
V4	0.0	

Effect	Temperature (ºC)	Pressure (kPa)	Δh (kJ/kg)	BPE (ºC)	Juice temp (ºC)	Dessin k (kW/m^2/K)	ΔT (ºC)	Sp evap (kg/m^2/h)	Area (m^2)	Evaporation (t/h)	Juice flow (t/h)	Brix	Steam req (t/h)
0	124.0	224.0	2191.3								500.0	13.50%	
1	114.9	167.8	2216.8	0.51	115.4	2.790	8.56	38.8	4840.1	187.7	312.3	21.62%	189.9
2	106.3	125.9	2240.3	0.93	107.3	2.214	7.65	27.2	3662.6	99.7	212.6	31.76%	100.8
3	99.4	98.7	2258.9	1.26	100.7	1.747	5.67	15.8	2332.4	36.8	175.7	38.41%	37.1
4	89.9	69.5	2283.8	1.95	91.9	1.243	7.55	14.8	2461.8	36.4	139.3	48.46%	36.8
5	65.0	24.8	2346.7	3.58	68.6	0.312	21.33	10.2	3476.0	35.5	103.8	65.00%	36.4