AUTOMATIC MEASUREMENT OF CRYSTAL SIZE DISTRIBUTION USING IMAGE PROCESSING

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Abstract

In sugar production the optimal control of sugar crystal growth during the crystallisation stages is essential for producing sugar crystals having the required size specification. The growth process is usually monitored using a variety of indirect measurement techniques in addition to visual inspection. Such indirect measurements are subjective and hence prone to operator variability. For this reason, a more quantitative automatic system is required. Software routines were developed for the automated measurement of crystal size using classical image analysis techniques.

Five algorithms have been evaluated to automatically determine the size distribution from images of crystal samples captured by a digital camera attached to a microscope. The accuracy of the segmentation depends on the technique used to mark every crystal in the image.

The parameters of interest which were calculated from the size distribution include the mean aperture (MA) and coefficient of variance (CV). Mathematical morphology has been used as a tool to develop the algorithms. The results obtained using these algorithms were compared with each other and with a manually clicked measurement. The mean aperture was measured with an error of less than 20%. The accuracy of the results mainly depended on the number of crystals counted and the degree of crystal overlap.

Keywords: MA, CV, crystal size distribution, image processing, grain size analysis, factory process

Introduction

Sugar production undergoes a series of steps, beginning with the sugarcane that arrives at the factory and ends with final product. The shredder crushes the cane into small pieces, after which the mills separate the juice which contains the sugar from the fibrous part. The juice is then clarified, which removes dirt and other impurities. The clarified juice then goes to the evaporation stage where most of the water is removed in a series of evaporators. This syrup is then transferred to a series of crystallisation pans where sugar crystals are grown from crystal seeds. The crystal growth is usually controlled by measuring conductivity or boiling point elevation, in addition to visual inspection. Such indirect monitoring can be compromised by operator fatigue and error. The visual inspection of the crystals, moreover, takes place off-line in a time intensive manner. Due to the fact that the quality of the sugar and the crystal growth rate depend on the crystal size distribution, a size monitoring and quantifying system is required. This work is aimed at designing and testing a system that measures the crystal parameters, such as mean aperture (MA) and coefficient of variance (CV), using classical

image processing techniques to automatically analyse a crystal image captured using a high resolution CCD camera attached to a microscope.

Grain size analysis

The sieve method for determining the size properties of raw and refined sugars is well established internationally for quality assessment (Miller and Beath, 2000). This method has several advantages and disadvantages. It is a relatively straightforward technique that can be implemented in a production environment and requires only low technology equipment to give acceptable reproducibility. It does, however, require a large sample and a relatively long preparation and measurement time. The major disadvantage of the sieving method is that the size distribution is characterised by a relatively small number of parameters, since typically from 1 to 8 sieves are used. This prevents the detailed determination of the size distribution. It is, therefore, difficult to accurately characterise a crystal population, using a small number of sieves, if the sugar contains, for example, a large percentage of conglomerates or agglomerates, or has a high 'fines' component due to crystal breakage. As described by Anon (2005), the apparatus for the raw sugar sieving method comprises a sample divider or riffle, top pan balance (which can measure to 0.01 g accuracy), woven wire cloth sieves (1700, 1180, 1000, 850, 600, 500, and 350 µm) and a mechanical shaker. The procedure involves sub-sampling until a sub-sample of about 100 g is obtained, which is then washed and dried to ensure that the crystals are well separated. The sieving procedure involves first weighting each sieve and the base pan to the nearest 0.01 g and assembling the sieves in descending order of aperture size. The measured sub-sample is then transferred to the top sieve and shaken for 15 minutes. The retained sugar is weighed and the balance is transferred to the next smallest sieve and shaken. This process is repeated. The values of the retained masses are converted to mean aperture (MA) and coefficient of variation (CV) using a calculation that assumes a model size distribution. Significantly, although 7 sieve sizes are recommended in the Anon (2005) specification, in other production environments (e.g. in refineries) a smaller number of sieves (e.g. 3) are used. This can be acceptable, provided the size distribution has a consistent shape.

According to Anon (2005), mean aperture (MA) is defined as the aperture (sieve size) that would retain 50% by mass of the sample, and the coefficient of variance (CV) is defined as the standard deviation of the distribution expressed as a percentage of MA. Thus,

$$CV(\%) = \frac{(f_{16} - f_{84})}{2 \times (MA)} \times 100$$

where f_{16} and f_{84} are the sieve sizes that retain 16% and 84% of the sugar sample.

Dalzeil *et al* (1999) described two types of grain size distributions from which the mean aperture (MA) and the coefficient of variance (CV) can be estimated, and compared these with the standard methods. The types of distributions are the mean-size-by-mass (volume) and the mean-size-by-number of the crystals. Mean-size-by-mass measures the cumulative mass of sugar retained by a given set of sieves used in the grain size estimation. Mean-size-by-number is typically based on the size analysis of a number of individual crystals using point-and-click image analysis methods. It was, however, stated by Peacock (2000) that MA and CV calculated using these two methods were not exactly equivalent, and recommended mean-size-by-number and its standard deviation for production purposes. In this research, however, mean-size-by-mass distribution is employed as it is more easily estimated from the projected area of the two-dimensional crystal images.

Technique to calculate MA and CV from crystal images

The question is, how is it possible to estimate MA and CV using classical image processing algorithms? In this work a mean-size-by-mass distribution has been employed. The volume distribution is related to the projected area and number of crystals identified (Dalzeil *et al*, 1999), i.e.

$$V \propto N(A)^{\frac{3}{2}}$$

Using this relation, one can estimate the percentage of mass which would be retained in a number of hypothetical bins if the sugar were to be sieved. Thirty equal size bins were used to be consistent with the procedures recommended by the Sugar Milling Research Institute (SMRI). Knowing the crystal size distribution, it is possible to calculate MA and CV using four techniques, namely (i) the powers method, (ii) the Rens method, (iii) the Rosin-Rammler-Sperling-Bennet (RRSB) method, and (iv) the Butler method (Anon, 1994). After assessing the techniques currently used in South Africa, Peacock (2000) recommended RRSB as a standard. After comparative study of those parameters estimated using Rens and RRSB methods, Schoonees (2002) confirmed that the average differences between the two calculations falls within the tolerance set by SMRI and recommended the use of the Rens method. The Rens method was thus employed to estimate CV and MA.

Rens method (Anon, 1994)

An empirical equation is used to describe the particle size distribution and is used to convert the cumulative percentage retained by different aperture sizes to a linear relationship. For each sieve aperture size, *d*, the cumulative percentage retained, *y*, is converted to a corresponding linear value, *z*, using the following empirical function:

For $y < 50\%$

$$z = -34.5\left[\sqrt[1.14]{\ln\frac{50}{y}} - e^{-0.18\,y}\right]$$

For $y = 50\%, \quad z = 0$

For $y > 50\%$

$$z = 34.5\left[\sqrt[1.14]{\ln\frac{50}{100-y}} - e^{-0.18(100-y)}\right]$$

The formula is only applied to values of *y* greater than 10% and less than 90%.

The linear relation between *d* and the corresponding calculated values *z* is then determined by regression analysis to give,

$$d = a + kz$$

where *a* and *k* are constants obtained from the regression analysis.

The aperture retaining 50% of the sugar is obtained when $y = 50$ and therefore $z = 0$, thus

$$d_{50} = a = MA$$

If d_{16} is the aperture size that retains 16% of the sugar, then

$$d_{16} = MA + kz_{16}$$

Since one standard deviation on both sides of the mean approximately represents 68% (34% on either side) of the data, the sieve size that retains 16% or 84% of the data (sugar crystals) represents one standard deviation away from the mean. The standard deviation of the particle size distribution is then given by $(MA - d_{16})$ and $(MA - d_{16}) = kz_{16}$ and consequently:

$$CV = \frac{kz_{16}}{MA} * 100$$

This method is employed to estimate MA and CV for grain size distribution in the following section.

Alternative crystal size determination techniques

Generally, alternative (non-sieving) crystal sizing techniques can be categorised into two major categories, i.e. image and non-image based. Imaging refers to those techniques which involve acquiring and analysing images of the crystals.

Non-image based techniques

Miller and Beath (2000) presented a comparison of crystal sizes measured using laser diffraction and the traditional sieve technique. This measurement may be made with crystals dispersed in a suitable suspending fluid such as methanol, or suspended in a parent growth solution and pumped through a flow cell from either a pilot scale apparatus or a factory pan. Sieves of sizes 420, 600, 850, 1000 and 1200 µm were used. The results showed a linear relationship between MA using the sieving technique, and the volume median diameter (assuming that the crystals were spherical) using laser diffraction. Other crystals properties such as the specific surface area and surface area mean diameter were calculated for samples from four sugar mills and compared with the sieve technique. It was, however, observed that linear regression analysis for these parameters showed poor correlations and large standard errors.

Schultz and Edye (2000) introduced an on-line device for the measurement of crystal content of massecuite. The system was developed using low resolution nuclear magnetic resonance (NMR) instrumentation and was based on measuring the contribution of the solid and liquid parts of the massecuite to the decay of the NMR decay signal. It was designed to provide the time history of the crystal content in the pans. A knowledge of the crystal content for a given crystallisation pan helped the controller take corrective action to optimise the pan capacity. The system, however, provided no direct measured value for the size of crystal, but only a graph of the percentage of crystal content as a function of time. Moreover, no comparison was made to verify the accuracy of the actual crystal content percentage.

Image based techniques

Palenzuela and Cruz (1996) have classified sugar crystal images based on spectral analysis and the use of neural networks. The spectrum shape parameters (spectrum maximum, high energy bandwidth, total spectral energy) were used as inputs to a neural network to classify the images in terms of the crystal homogeneity, size distribution, etc. The results of the classification were purely qualitative, indicating the presence or absence of the above-

mentioned situations. There was, thus, no numerical value given to quantify the size of the crystals in the image.

Dalziel *et al* (1999) introduced a classical image analysis based system for crystal sizing. An image based algorithm first sorted the objects according to their grey scale values. Depending on their roundness, unwanted objects, including clusters of crystals, side-on crystals and bubbles were ignored. The problem of touching and overlapping crystals was addressed semi-automatically using a splitting function. However, not all overlapping and touching crystals could be separated using the same function and in these cases their boundaries were traced manually. Various properties of the region including area, perimeter, maximum linear dimension, aspect ratio, and roundness were estimated. Other parameters such as the volume equivalent size and projected area were calculated and used to compute the crystal size distribution. MA and CV were calculated using the imaging system and sieve technique for three dry sugar samples taken from sugar mills. There was, however, no ground truth to confirm the accuracy of those values. An interesting result shown in the paper was that the ratio of L (the diameter of a sphere that has the same volume as the crystal) to $\sqrt{A}$ (square root of projected area) for Australian sugar which was constant, i.e.

$$\frac{L_v}{\sqrt{A}} = 0.97 \pm 0.01.$$

This ratio was used to convert a two-dimensional projected area size into a three-dimensional volume equivalent size.

Peacock (1998) suggested the possible application of neural networks in the sugar industry for process control. Mhlongo and Alport (2002) developed a crystal sizing technique using an artificial neural network after first performing a wavelet analysis of crystal images. The wavelet coefficients were used as inputs to a Multi-Layer Perceptron (MLP). An analysis of the percentage error in crystal size indicated that Daubechies wavelet coefficient performed better than the Haar, Symlet and Coiflet wavelets. Test results on 144 pan images showed fairly good correlation, with a mean percentage error of 9%. Predicted mean size and actual mean size for crystal images obtained from a laboratory crystaloscope immersed in a glycerin solution showed a mean percentage error of 12.9%.

Methods

Imaging system

The imaging system used comprised a high resolution CCD camera attached to microscope. The 8x magnified microscope images were captured and digitised to give a 576x768 pixel color image at a resolution of 24 bits. The pixel separation, for 8x magnification, corresponded to 8.85 μm in the horizontal and 16.49 μm in the vertical direction.

Image acquisition

For the purpose of this work, 25 dry sugar samples obtained from the Huletts refinery were used. Crystals were randomly spread on a microscope slide and images (each containing about 100 crystals) were captured. These images contained crystals having a variety of spatial configurations such as clustering, overlapping or just touching. The crystals were purposely not separated to ensure that the images represented typical configurations that could be encountered if the images were taken from the crystallisation pan in a production environment. The analysis was performed using MATLAB.

Image processing

The flow diagram in Figure 1 depicts the steps used to segment the crystal images. The acquired RGB color image is converted to a grey level image by averaging the intensity levels in each of the R, G and B channels. This is followed by histogram equalisation to spread the pixel values to all possible grey levels. An automatic thresholding function is applied to binarise the greyscale images. The threshold value is computed using a method described by Otsu (1979) which is based on comparing the between class variance and, finally, by choosing the one which gives the maximum between class variance. This segments the foreground from the background, except for some foreground pixels which are misclassified as background. Filling the holes using morphological operators will binarise the image, with the objects having a pixel intensity of one and the background that of zero. A number of different marking techniques were evaluated to uniquely identify individual single crystals:

- The foreground and background marker (FB) uses a regional minimum to mark the foreground. This is achieved by combining gray scale morphological erosion and dilation with gray scale reconstruction to maintain the original size of the crystals. In some images the background as well as the crystals were marked. This is undesirable because the application of the watershed algorithm on those images introduces a region which does not correspond to a crystal. Therefore, the requirement to mark only the crystals is crucial for accurate segmentation.

- The ultimate erosion (UE) algorithm uses the last connected components to mark the crystals. The last connected component is obtained by repeatedly eroding the binary image using a structural element, but stopping one step before the whole object disappears. This technique has been employed to segment coffee bean images (Vincent, 1993).

- The distance transform (DT) method has been used in biological studies to mark clustered nuclei (Malpica *et al*, 1997). It can segment round shaped objects efficiently because the center of the object is the only point, equidistance from the background, marked as a minimum. Non-round objects are marked by more than one minima, which results in over-segmentation. This can introduce erroneous data when estimating some properties of a region, such as its area.

- In some crystals, using the above techniques individually can produce crystals with more than one marker. This leads to an oversegmentation of the crystals when the watershed algorithm is applied. It is necessary to minimise such occurrences, and to mark each object with only one marker. This was achieved fairly successfully by combining two of the above techniques: FB+UE or UE+DT. This also reduced miss-marking of the background.

Finally, a watershed function was applied to determine the crest lines which separate the objects from each other and from the background.

Watershed segmentation is a morphological algorithm which permits the detection of crest lines in images (Beucher, 1992). Considering the grey-tone image as a topographic surface, water falling on it will flow down the walls of the catchment basin corresponding to each minimum. The points where the water can flow down either one of the two sides, form the crest lines which are to be detected. An efficient implementation based on immersion

simulation has been presented by Vincent and Soille (1991). This approach may be described as follows. If a hole is drilled in each minimum and filled progressively with water, the watersheds would correspond to the line of the points where water coming from two different basins would meet. The concept of immersion can be implemented by making use of a hierarchical queue, with priorities defined by the grey level of image points. Points are recursively inserted and extracted from the queue, in an order defined by their grey levels. In this way, the lowest grey levels are processed before the highest ones.

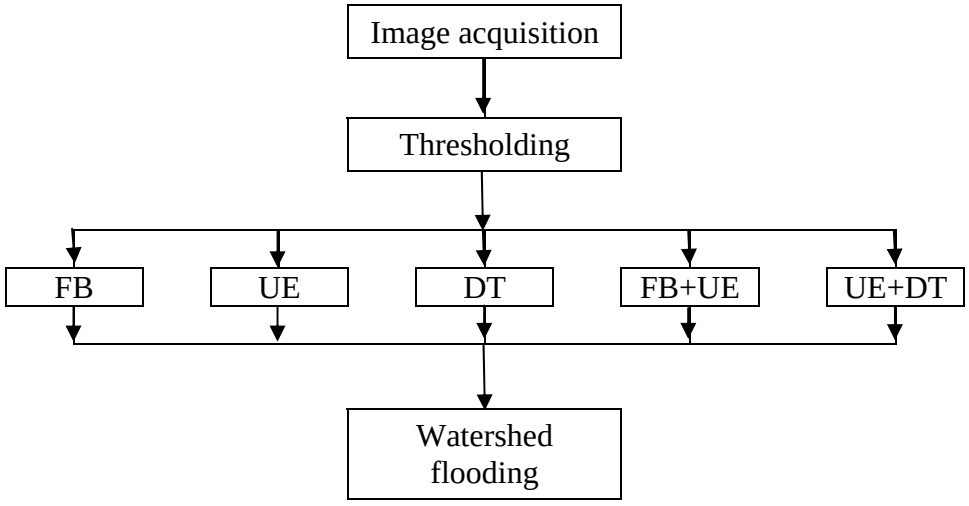

Figure 1. The steps used to segment the crystal images. The acquired image is binarised using Otsu's (1979) thresholding method, and then marked using foreground / background markers (FB), ultimate erosion (UE) and distance transform (DT), or a combination of these techniques. Finally, watershed flooding was applied to determine the crest lines which separate the objects from each other and from the background.

To be able to apply this algorithm to detect lines dividing clustered objects, two main actions have to be carried out:

- the original image has to be transformed into a different image, where following the topographical 'crest lines' correspond to the original image object boundaries and their inner and outer parts to 'valleys', and

- singular markers are to be defined on every valley of the transformed image as starting points of the flooding process.

An incorrect choice of the image transformation usually leads to incorrect segmentation, and failure in assigning a single marker to each valley of the transformed image leads to over-segmentation.

Figure 2 indicates how well these algorithms segment a sample crystal. When the distance transform was combined with ultimate erosion (DT+UE), this gave the best segmentation, since the contour most closely coincides with the visible border. Foreground and background (FB), ultimate erosion (UE) and their combination (UE+FB), however, underestimated the sample crystal.

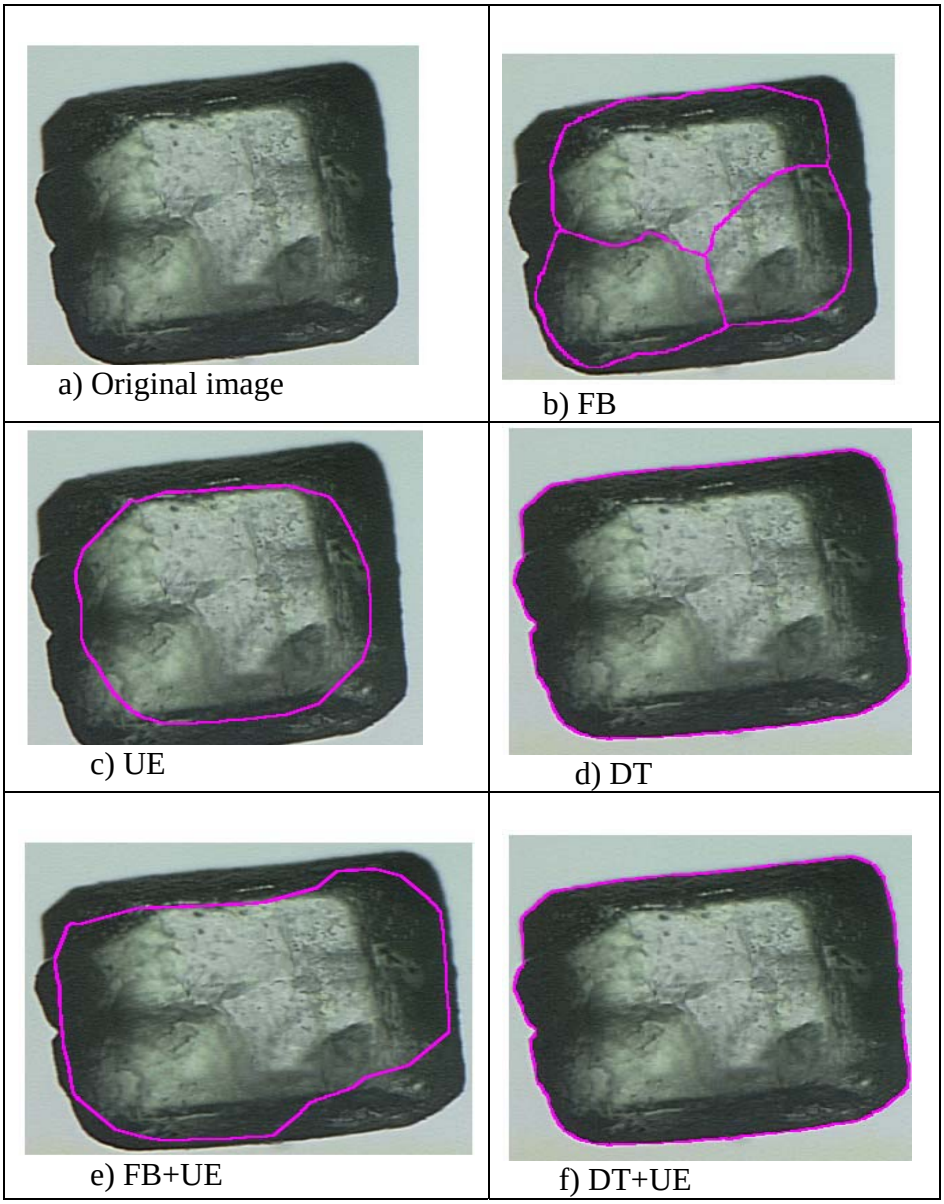

Figure 2. Examples of how well the algorithms segment a sample crystal (a) from the background. The distance transform (DT) gives the best result when combined with ultimate erosion (UE). The individual algorithms FB (b), UE (c) and DT (d), in addition to ultimate erosion in combination with foreground and background marker (FB+UE), underestimate the crystal size.

Results

Verification of crystal sizing using sieving

A clicker software program has been written to allow the individual boundary of crystals to be manually traced using the mouse cursor. Although the use of this program is tedious, it is considered to offer the best ground truth against which the automatic algorithms and sieving techniques can be benchmarked.

The MA and CV of 25 sugar samples were compared with the manually clicked technique. Figure 3 indicates the number of crystals segmented using manual clicking and the UE+DT algorithm. In almost all images, the number of crystals is underestimated by the algorithm.

One reason for this is that fragmented small crystals are removed by the morphological operators. In some images the crystals touching the edge of the image were excluded by the segmenting algorithms, whilst some of them were included when the crystals were manually clicked. However, it could be argued that if some crystals are randomly removed from the images by the automatic algorithm, then the distribution would not be affected very much, and so it is still meaningful to compare the MA and CV obtained by the segmentation algorithms and the clicker program.

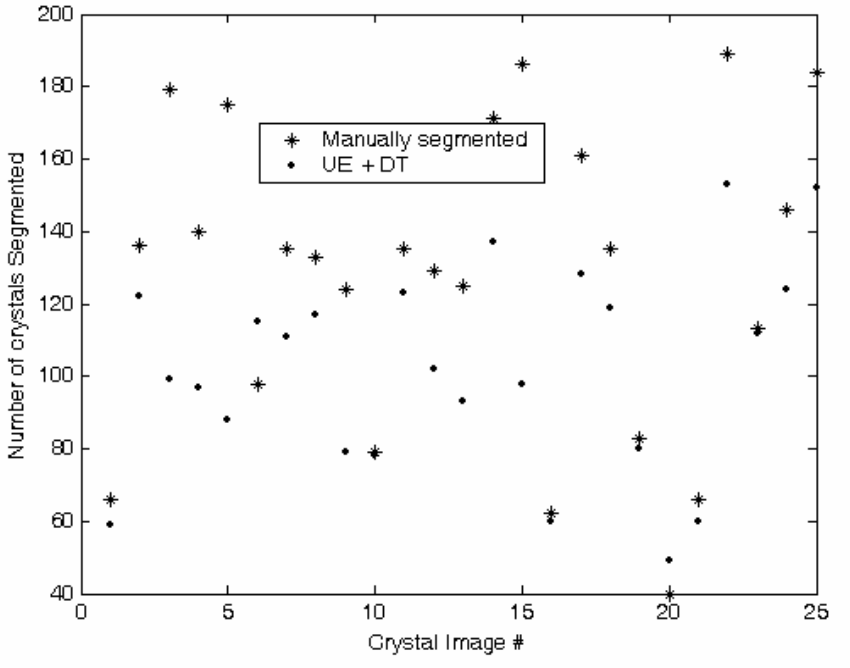

Figure 3. This graph shows a comparison of the number of crystals segmented using ultimate erosion plus distance transform (UE+DT) and manually for each of the 25 crystal images. In most images the number of crystals identified by the algorithm is less than the manual method. One reason for this was that the algorithm rejected small fragmented crystals.

Evaluation of the accuracy of the segmentation algorithm

Three algorithms for segmenting the crystal images have been evaluated. These include the foreground and background marker (FB), ultimate erosion (UE) and the distance transform (DT) methods. In addition, efforts were made to evaluate various combinations of these algorithms to enhance the accuracy. A measure of the accuracy can be defined as the average percentage error, ε,

$$\varepsilon(\%) = \left(\frac{\sum_{i=1}^{N} \frac{|P_i - C_i|}{C_i}}{N} \right) \times 100\%$$

where *P* is the MA of the i-th image using one or a combination of the three algorithms, C the MA determined by manually clicking and N is the number of images. Table 1 shows the values of ε_{MA} for the different algorithms. The error achieved by applying UE+DT is ε_{MA} = 5.45% and is significantly better than that achieved by the other algorithms.

Table 1. Average percentage error of the mean aperture, ε_{MA}, calculated after applying the segmentation algorithms background/foreground marker (BF), ultimate erosion (UE) and distance transform (DT) and the two combinations BF+UE and UE+DT.

	BF	UE	DT	BF+UE	UE+DT
ε_{MA}(%)	48.52	33.42	17.19	16.45	5.45

Because the combination of ultimate erosion and the distance transform (UE+DT) produced the smallest mean error, it was used for the remainder of the analysis.

Figure 4 shows the predicted mean area versus the mean area calculated using manual clicking from crystal images. The scatter plot is shown together with the best fitted line and the corresponding linear regression coefficient. The best fit straight line has a slope of 1.0 and has a small offset of 14.5 pixels. There are a number of outliers. Upon closer investigation of those images, it was found that they either contained a number of overlapping or fragmented crystals which were included in the manual clicking but rejected by the algorithm. The mean percentage error, ε, was 9.63%. This is lower than the result of 12.9% reported by Mhlongo and Alport (2002). However, for a more meaningful comparison of these two algorithms, it would be preferable to test them on the same sets of images.

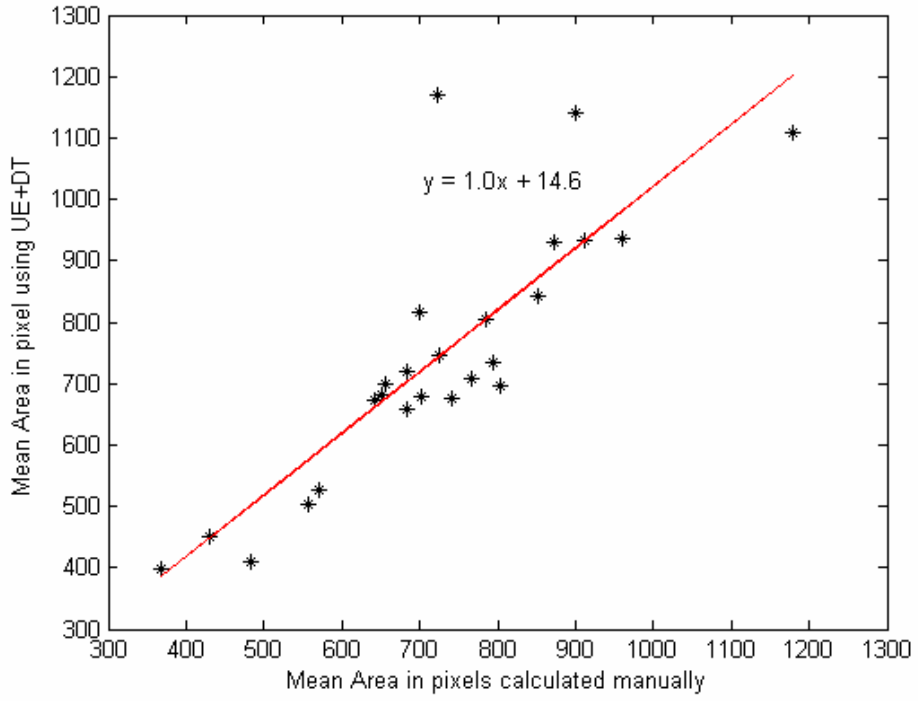

Figure 4. A scatter plot for the mean area calculated for all the crystals in each of the 25 images. On the vertical axis is the mean area predicted using ultimate erosion plus distance transform (UE+DT) and on the horizontal axis is the mean area calculated manually by clicking the individual crystals in each image.

Mean aperture for the crystal size distributions, shown in Figure 5, was estimated using the Rens method. To find MA from the images, the crystal areas were computed using UE+DT and this was converted to a mass distribution (as all crystals have the same density, by volume) and then to the percentage of crystals retained (cumulative). The mass distribution was computed for 30 equally spaced bin sizes. MA was then computed by applying the Rens

method for each crystal image. Fairly good correlation was observed, with regression coefficients of 0.85 and 126.8.

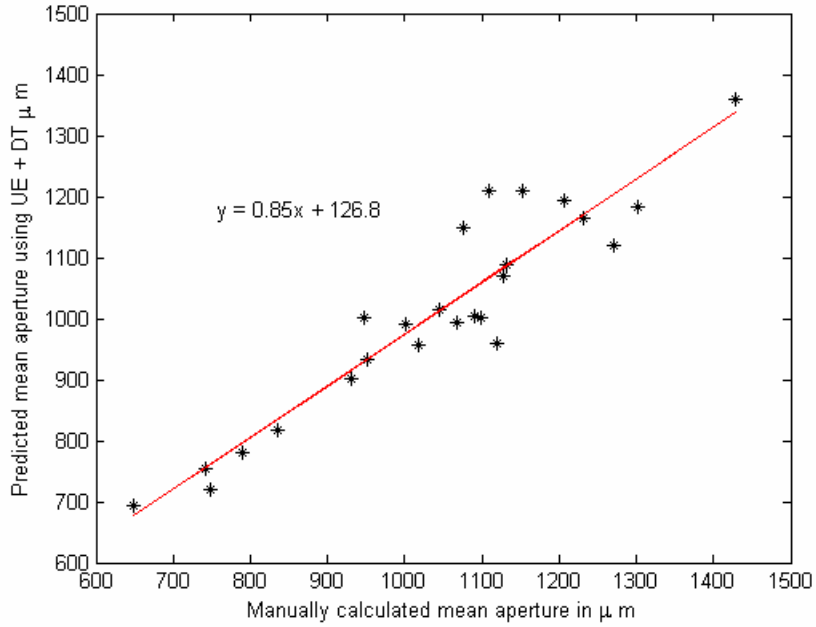

Figure 5. Scatter plot of the Mean Aperture for each of the 25 crystal images. The values on the vertical axis were calculated using ultimate erosion plus distance transform (UE+DT), and the horizontal values were calculated by manually clicking the individual crystals. The best-fit straight line has coefficients of 0.85 and 126.8.

However, as shown in Figure 6, CV is poorly estimated and there is a much larger scatter of the points. Thus, it appears that the crystals that are being removed by the DT+UE algorithm are affecting the shape of the distribution function. This problem required further investigation.

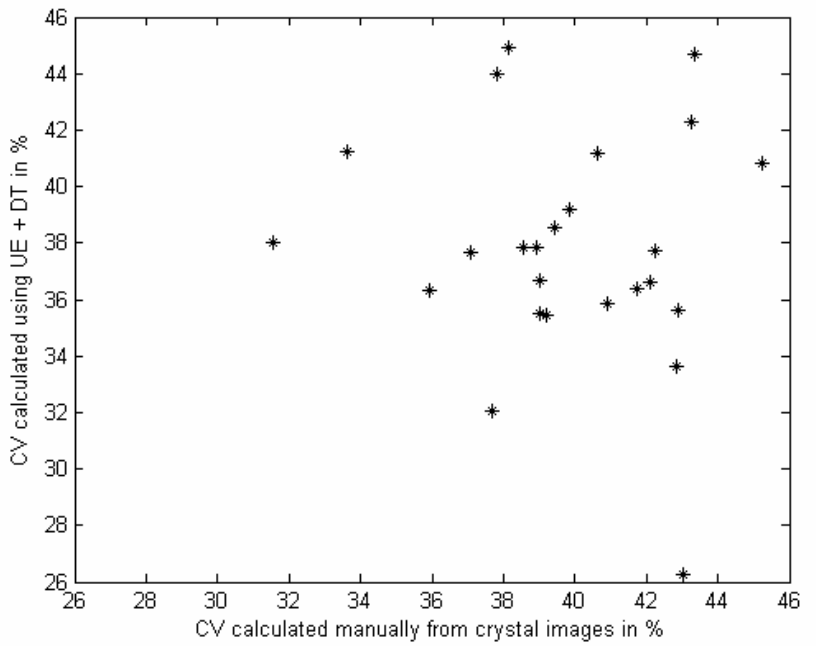

Figure 6. Scatter plot for coefficient of variance. With the vertical axis represents predicted value and the horizontal axis represents the one calculated manually from crystal images.

Conclusion and Discussion

Results of using an automated classical image processing technique to estimate the crystal size parameters MA and CV as a possible alternative to the sieving technique that is used in the production environment, are reported. Although a number of segmenting algorithms were evaluated, it was found that a combination of ultimate erosion and the distance transform (UE+DT) was the most accurate in finding the crystal boundaries. Watershed segmentation was employed to segment clustered, just touching and overlapping crystals. Using dry crystal samples obtained from a refinery, the MA and CV were calculated for each of 25 sample images using the Rens method after applying the UE and DT algorithms. These results were compared with those obtained by manual clicking on the crystal boundaries. MA showed a fairly good correlation with an average percentage error of 6%. This result is slightly better than the earlier one reported by Mhlongo and Alport (2002) using wavelets and neural networks. However, there was more scatter in the CV results, possibly indicating that the crystals that were eliminated by the UE+DT algorithm (either at the edge of the image, too fragmented, overlapping or over-segmented) was having an effect on the crystal size distribution. Further work is required to reduce these errors. Taking multiple images of each sample and automatically eliminating edge crystals can increase the total number of crystals analysed in each sample. Overlapping crystals can be identified using a curvature measure and optimisation of the morphological operators can retain fragmented crystals.

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